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C O N T E N T S

Vector-valued nonuniform multiresolution analysis on positive half line

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Multiresolution analysis (MRA) is an important tool since it provides a natural framework for understanding and constructing discrete wavelet systems. In his paper, Mallat first formulated the remarkable idea of multiresolution analysis (MRA) that deals with a general formlism for construction of an orthogonal basis of wavelet bases. Any compactly supported wavelet must come from a MRA.

The concepts of wavelet and multiresolution analysis has been developed to many different set ups. Gabardo and Nashed have studied nonuniform multiresolution analysis based on the theory of spectral pairs. Farkov has extended the notion multiresolution analysis on locally compact Abelian groups and constructed compactly supported orthogonal p-wavelets on $L^2(\mathbb{R}^+)$. Xia and Suter introduced vector-valued multiresolution analysis and orthogonal vector valued wavelets. Meenakshi, Manchanda and Siddiqi have generalized the concept of vector-valued multiresolution analysis to vector-valued nonuniform multiresolution analysis.

In this paper, we have considered the vector-valued nonuniform multiresolution analysis on positive half-line. The associated subspace V_0 of $L^2(\mathbb{R}^+, \mathbb{C}^N)$ has an orthonormal basis, a collection of translates of vector-valued function φ of the form $\{\varphi(x \ominus \lambda)\}_{\lambda \in \Lambda^+}$ where $\Lambda^+ = \left\{0, \frac{r}{N}\right\} + \mathbb{Z}^+$, where $N \geq 1$ is an integer, and r is an odd integer such that r and N are relatively prime, and $\mathbb{Z}^+$ is the set of non-negative integers. We obtain the necessary and sufficient condition for the existence of associated wavelets.

A relation between distributions of the distance and the random chord in a convex domain

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Let $\mathbf{D}$ be a bounded convex domain in the Euclidean plane, with the area $\|\mathbf{D}\|$ and the perimeter $|\partial\mathbf{D}|$. Let P_1 and P_2 be two points chosen at random, independently and with uniform distribution in $\mathbf{D}$. Our aim is to find the distribution function $F_\rho(x)$ of $\rho(P_1, P_2)$. By definition,

$$F_\rho(x) = P((P_1, P_2) \in \mathbf{D} : \rho(P_1, P_2) \leq x) = \frac{1}{\|\mathbf{D}\|^2} \iint\limits_{\{(P_1,P_2):\rho(P_1,P_2)\leq x\}} dP_1\, dP_2, \quad (1)$$

where dP_i, $i = 1, 2$ is an element of the Lebesgue measure in the plane.

From the expression of the area element in polar coordinates we have $dP_1\, dP_2 = r\, dP_1\, dr\, d\varphi$, where φ is the angle between the line through the points P_1, P_2 and the reference direction in the plane. If we leave r fixed, then $dP_1\, d\varphi$ is the kinematic density for the segment P_1P_2 of length r. Therefore, we can rewrite (1) in the form:

$$F_\rho(x) = \frac{1}{\|\mathbf{D}\|^2} \int_0^x r\, K(\mathbf{D}, r)\, dr, \quad (2)$$

where $K(\mathbf{D}, r)$ is the kinematic measure of all oriented segments of length r lying inside $\mathbf{D}$. Therefore, using (2) we obtain a relationship between the density function $f_\rho(x)$ of $\rho(P_1, P_2)$ and the kinematic measure $K(\mathbf{D}, r)$:

$$f_\rho(x) = \frac{x\, K(\mathbf{D}, x)}{\|\mathbf{D}\|^2}. \quad (3)$$

In the paper [1], a formula for the kinematic measure $K(\mathbf{D}, r)$ of sets of segments with constant length r entirely contained in $\mathbf{D}$ is obtained. The obtained formula in [1] allows to calculate the mentioned kinematic measure $K(\mathbf{D}, r)$ by means of the chord length distribution function of $\mathbf{D}$. We transform (3) to a more suitable for applications form:

$$f_\rho(x) = \frac{1}{\|\mathbf{D}\|^2}\left[2\pi\, x\, \|\mathbf{D}\| - 2\, x^2\, |\partial \mathbf{D}| + 2\, x\, |\partial \mathbf{D}| \int_0^x F_{\mathbf{D}}(u)\, du\right], \qquad (4)$$

where $F_{\mathbf{D}}(\cdot)$ is the chord length distribution function of the domain $\mathbf{D}$. Note that if we know the explicit form of the chord length distribution function for a domain, using (4) we can calculate the density function $f_\rho(x)$. In [2] the explicit form of the chord length distribution function is given for any regular polygon.

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One hundred years of uniform distribution theory. Hermann Weyl's foundational paper of 1916

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In 1916 Hermann Weyl published his paper "Über die Gleichverteilung von Zahlen mod. Eins", which established the theory of uniform distribution modulo one as a proper mathematical discipline and connected it with various other mathematical topics, including Fourier analysis, number theory, numerical analysis and probability theory. Weyl's paper was of seminal importance for the development of mathematics in the twentieth century, and it is astonishing that the seed for so many later developments can already be found in this paper. In this talk we give an outline on the history of the development of Weyl's paper, its content, subsequent developments and on remaining open problems.

On orders of trigonometric widths of the Nikol'skii – Besov class in a Lorentz space

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Let $\overline{x} = (x_1, ..., x_m) \in \mathbb{T}^m = [0, 2\pi]^m$, $I^m = [0, 1)^m$ and let $\tau, p \in [1, +\infty)$.

We will denote by $L_{p,\tau}(\mathbb{T}^m)$ the Lorentz space of Lebesgue - measurable functions $f(\overline{x})$ of period 2π in each variable such that

$$\|f\|_{p,\tau} = \left\{ \frac{\tau}{p} \int_0^1 \left(\int_0^t f^*(y) dy \right)^{\tau} t^{\tau(\frac{1}{p}-1)-1} dt \right\}^{\frac{1}{\tau}} < +\infty,$$

where $f^*(y)$ is the non-increasing rearrangement of the function $|f(2\pi\bar{x})|$, $\bar{x} \in I^m$ (see [1], pp. 83, 197).

Let $a_{\overline{n}}(f)$ be the Fourier coefficient of $f \in L_1(\mathbb{T}^m)$ with respect to the multiple trigonometric system. Then we set $\sigma_s(f, \overline{x}) = \sum\limits_{\overline{n} \in \rho(s)} a_{\overline{n}}(f) e^{i\langle \overline{n}, \overline{x} \rangle}$, where $\langle \bar{y}, \bar{x} \rangle = \sum\limits_{j=1}^{m} y_j x_j$, and for $s = 0, 1, 2, ...$,

$$\rho(s) = \left\{ \overline{k} = (k_1, ..., k_m) \in Z^m : \quad [2^{s-1}] \le \max_{j=1,...,m} |k_j| < 2^s \right\}.$$

Consider the Nikol'skii – Besov class: for $1 < p < \infty$, $1 < \tau < \infty$,

$1 \leq \theta \leq \infty$ and $r > 0$, we denote

$$B^r_{p,\tau,\theta} = \left\{ f \in L_{p,\tau}\left(\mathbb{T}^m\right) : \left(\sum_{s \in \mathbb{Z}_+} 2^{sr\theta} \left\| \sigma_s(f) \right\|^{\theta}_{p,\tau} \right)^{\frac{1}{\theta}} \leq 1 \right\}.$$

The main aim of the present talk is an estimate of the order of trigonometric widths of Nikol'skii–Besov classes in the metric of the Lorentz space.

Theorem. *If* $1 < p < 2 < q < \frac{p}{p-1}$, $1 < \tau_1, \tau_2 < +\infty$, $1 \leq \theta \leq \infty$, $r > m$, *then*

$$d^T_n(B^r_{p,\tau_1,\theta}, L_{q,\tau_2}) \asymp n^{-\frac{r}{m}+\frac{1}{p}-\frac{1}{2}}.$$

If $1 < p < q < 2$, $r > m(\frac{1}{p} - \frac{1}{q})$, *then*

$$d^T_n(B^r_{p,\tau_1,\theta}, L_{q,\tau_2}) \asymp n^{-\frac{r}{m}+\frac{1}{p}-\frac{1}{q}}.$$

If $1 < p < 2 < q < \infty$, $m(\frac{1}{p} - \frac{1}{q}) < r < \frac{m}{p}$, *then*

$$d^T_n(B^r_{p,\tau_1,\theta}, L_{q,\tau_2}) \geq C n^{-\frac{q}{2}(\frac{r}{m}+\frac{1}{q}-\frac{1}{p})}.$$

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Optimal uniform approximation on the angle by harmonic functions

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In this talk we discuss the problem of optimal uniform approximation on the angle $\Delta_\alpha = \{z \in \mathbb{C} : |\arg z| \leq \alpha/2\}$ by harmonic functions. The approximable function is harmonic on the interior of Δ_α and satisfies some conditions on the boundary of Δ_α. The estimations of the growth of the approximating harmonic functions on $\mathbb{R}^2$ depend on the growth of the approximable function on Δ_α and its smoothness on the boundary of Δ_α.

The problem of uniform approximation on the sector by entire functions was investigated by H. Kober [1], M.V. Keldysh [2], Mergelyan [3], N. Arakelian [4] and other authors. The analogue problem in the case for meromorphic functions was discussed in [5].

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On approximation of continuous functions by Fourier-Haar sums and Haar polynomials

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Golubov [1] found a sharp constant in direct approximation theorem for a classical Haar basis on a line. He also proved the inverse theorem for this basis.

We consider a multivariate Haar system constructed in the framework of wavelet theory, so called separable Haar basis. After periodization of this system one has a periodic orthonormal basis which can be enumerated in a natural way.

For such basis we prove direct and inverse approximation theorems. The direct theorem is a sharp Jackson type inequality. The estimate of the best approximation E_n is given by a linear combination of partial moduli of continuity, where the coefficients are sharp constants.

Also, for periodic continuous functions of two variables a sharp estimate of the deviation from Fourier-Haar sums in terms of the modulus of continuity is obtained.

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Zonoids with equatorial characterization

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In [1] it was found a sufficient condition for a centrally symmetric convex body to be a zonoid. The condition was formulated in terms of characteristics of equators of the body. Using this condition one can define a class of convex bodies admitting equatorial characterization (see [2]). Also it was proved that ellipsoids belong to that class.

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Sparse subsets of Rademacher chaos in symmetric spaces

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As usual, the *Rademacher functions* are defined as follows: if $0 \leqslant t \leqslant 1$, then $r_n(t) := \operatorname{sign}(\sin(2^n \pi t))$, $n = 1, 2, \ldots$ By the *Rademacher chaos of order* $d \in \mathbb{N}$ we mean the set of all functions of the form $r_{i_1 i_2 \ldots i_d}(t) := r_{i_1}(t) \cdot r_{i_2}(t) \cdot \ldots \cdot r_{i_d}(t)$, where $i_1 > i_2 > \ldots > i_d \geqslant 1$.

The following concept of the combinatorial dimension was introduced by R. Blei. A set $S \subset \mathbb{N}^d := \mathbb{N} \times \mathbb{N} \times \ldots \times \mathbb{N}$ (d factors) is said *to have* *combinatorial dimension* α if

1) for arbitrary $\beta > \alpha$ there exists $C_\beta > 0$ such that for every collection of sets $A_1, A_2, \ldots, A_d \subset \mathbb{N}$, $|A_1| = |A_2| = \ldots = |A_d| = m$ ($|A|$ is the number of elements of a set A), we have

$$|S \cap (A_1 \times A_2 \times \ldots \times A_d)| < C_\beta m^\beta;$$

2) for any $\gamma < \alpha$ and $k \in \mathbb{N}$ there are sets $A_1, A_2, \ldots, A_d \subset \mathbb{N}$, $|A_1| = |A_2| = \ldots = |A_d| = m > k$, for which

$$|S \cap (A_1 \times A_2 \times \ldots \times A_d)| > m^\gamma.$$

We will discuss relations between basic properties of subsequences of the Rademacher chaos in symmetric function spaces and combinatorial dimension of corresponding sets. In particular, the following result gives necessary and sufficient conditions under which such a subsequence, generated by a set of the maximal combinatorial dimension, is unconditional basis in a symmetric space.

Theorem 1. *Let X be a symmetric space on $[0,1]$, $d \in \mathbb{N}$, $d \geqslant 2$, and let the set $S \subset \triangle^d$ have combinatorial dimension d. The following conditions are equivalent:*

1) $\{r_{i_1 i_2 \ldots i_d}\}_{(i_1, i_2, \ldots, i_d) \in S}$ is an unconditional basis sequence in X;

2) the sequence $\{r_{i_1 i_2 \ldots i_d}\}_{(i_1, i_2, \ldots, i_d) \in S}$ is equivalent in X to the canonical basis in ℓ_2;

3) $X \supset G_{2/d}$, where $G_{2/d}$ is the closure of L_∞ in the Orlicz space $ExpL^{2/d}$ generated by the function $M(u) \sim \exp(u^{2/d})$.

This is a joint work with K.V. Lykov.

On harmonic conjugates in weighted Dirichlet spaces of quaternion-valued functions

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Weighted Dirichlet spaces of Clifford valued functions are intensively studied in the recent years by several authors as M. Shapiro, Gürlebeck, Malonek, Cnops, Delanghe, Brackx, Bernstein, Kähler, Tovar, Resendis and others, see [1] and references therein. Various characterizations of weighted Dirichlet spaces of Clifford valued functions are given in their papers.

In this note, we pose and solve the harmonic conjugation problem in weighted Dirichlet spaces of quaternion-valued functions on the unit ball in $\mathbb{R}^3$. A special constructive approach ([2]) for the generation of harmonic conjugates is applied to find a monogenic function with values in the reduced quaternions and a given scalar part.

Our question is: If the given harmonic function belongs to a certain function space, specifically to the weighted Dirichlet space, can we conclude that the conjugate harmonic functions and so constructed monogenic function belong to the same space? The answer is "yes" for some appropriate indices. Earlier, similar results for weighted Hardy and Bergman spaces of quaternion-valued functions are obtained in [3], [2].

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On the usage of 2-node lines in n-poised sets

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Denote by Π_n the space of bivariate polynomials of total degree at most n, whose dimension is:

$$N := \dim \Pi_n = \binom{n+2}{2}.$$

An n-poised set in two dimensions is a set of nodes admitting unique bivariate interpolation with polynomials from Π_n.

Consider a set of N distinct nodes

$$\mathcal{X} = \{(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)\}$$

A polynomial $p \in \Pi_n$ is called an n-fundamental polynomial for a node $A = (x_k, y_k) \in \mathcal{X}$ if

$$p^{\star}_{A,\mathcal{X}}(x_i, y_i) = \delta_{ik}, i = 1, \dots, N,$$

where δ is the Kronecker symbol.

We say that a node $A \in \mathcal{X}$ uses a line ℓ, if ℓ is a factor of the fundamental polynomial $p^{\star}_{A,\mathcal{X}}$.

Proposition. *Let $\mathcal{X}$ be any n-poised set and ℓ is a line passing through exactly 2 nodes of $\mathcal{X}$. Then ℓ can be used at most by one node of $\mathcal{X}$.*

This statement for the special case when $\mathcal{X}$ is a GC_n-set is proved in [1]. The node set $\mathcal{X}$ is called GC_n-set, if the fundamental polynomial of each node is a product of n linear factors.

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Embedding theorem for $W^{1,n}(D)$ for sets of an arbitrary measure

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Let $D \subset R^n$ be an open set, $(n \geq 2)$, μ be the Lebesgue measure in R^n, $S(\mu; D)$ be the space of measurable functions $u : D \to R$ and let $X \subset S(\mu, D)$ be a symmetric space. Also let C_n be the volume of the unit ball in R^n, and $\rho(D) = \left(\frac{\mu(D)}{C_n}\right)^{1/n}$.

We denote by $W^{1,p}(D, X)$, $(1 \leq p < \infty)$ the closure of $C_0^\infty(D)$ in the Sobolev norm

$$\|u|W^{1,p}(D, X)\| = \|\nabla u|L^p\| + \|u|X\|,$$

(the symbol $\|y|Y\|$ indicates the norm of the element y in the space Y.)

The main problem, which goes back to the pioneering paper of S.L. Sobolev, is to find the "minimal" space Y, for which the embedding

$$W^{1,p}(D, X) \subseteq Y$$

holds and also to estimate the constant of this embedding.

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We define the function $\psi_{R,n} : (0,\rho) \to R_+$ by

$$\psi_{\rho,n}(t) = nC_{n-1}^{1/n} \begin{cases} (ln\frac{\rho}{t})^{-1/n'}, & \text{if } t \in (0, \rho e^{-1/n'}), \\ (\frac{1}{n'})^{-1/n'}, & \text{if } t \in [\rho e^{-1/n'}, \rho) \end{cases}$$

and define the function $w_{\rho,n} : (0,\rho) \to R_+$ by equality $w_{\rho,n}(t) = nC_{n-1}^{1/n} \cdot \frac{1}{t} \cdot (ln\frac{2\rho}{t})^{-n}$.

Theorem. *Fix an open set D. Then for any u from the unit ball of $W^{1,n}(D,X)$, the following sharp inequalities are valid:*

$$\sup_{\rho \in (0,\mu(D))} \{\|(u^*(.) - u^*(\rho))\chi(0,\rho)|\widetilde{M(\psi_{\rho,n})}\| +$$

$$\|u^*(\rho)\chi(0,\rho) + u^*(.)\chi(\rho,\mu(D))|X\|\} \le \|u|W^{1,n}(D,X)\|,$$

$$\sup_{\rho \in (0,\mu(D))} \{2\|(u^*(.) - u^*(\rho))\chi(0,\rho)|M(\psi_{\rho,n})\| +$$

$$\|u^*(\rho)\chi(0,\rho) + u^*(.)\chi(\rho,\mu(D))|X\|\} \le \|u|W^{1,n}(D,X)\|,$$

$$\sup_{\rho \in (0,\mu(D))} \{\|(u^*(.) - u^*(\rho))\chi(0,\rho)|L^n_{w_{\rho,n}}\| +$$

$$\|u^*(\rho)\chi(0,\rho) + u^*(.)\chi(\rho,\mu(D))|X\|\} \le 2\|u|W^{1,n}(D,X)\|.$$

Here we use the following notations:

$$\|u|\widetilde{M(\varphi)})\| = \sup_{\alpha>0} \alpha\varphi(\lambda(u,\alpha)) = \sup_{\alpha>0} u^*(\alpha)\varphi(\alpha),$$

$$\|u|M(\psi)\| = \sup_{0<t<a} \frac{\psi(t)}{t}\int_0^t u^*(s)ds = \sup_{D\subseteq D} \left\{\frac{\psi(\mu(D))}{\mu(D)}\int_D |u(s)|d\mu(s)\right\},$$

$$\|u|L^n_{w_{\rho,n}}\| = \left\{\int_0^\rho (u^*(s)w_{\rho,n}(s))^n ds\right\}^{1/n}.$$

The abstract form of the Kolmogorov's theorem on divergent trigonometric series

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The aim of this talk is to obtain the broadest possible generalization of the fundamental theorem of A. N. Kolmogorov on the existence of almost everywhere divergent Fourier-Lebesgue trigonometric series. By developing the author's method concerning the averaging on the supports of delta functions, here we obtain an abstract form of the Kolmogorov's theorem, which holds true for any bounded biorthogonal system of complex-valued functions defined on arbitrary measure space. We obtain exact logarithmic lower estimate for the majorant of the partial sums segments of two conjugate Fourier series, taken by the family of delta functions with appropriately chosen supports. The obtained abstract theorem is applied to construct divergent Fourier-Lebesgue series by biorthogonal systems of complex-valued functions, defined on metric spaces or on topological groups. This new, complex version enables to widen the range of applications of this theorem and gives a possibility to use it to study the systems of characters, and also the biorthogonal systems consisting of functions of complex variables. To achieve this, it was necessary to essentially complicate and revise the real-valued construct.

Lebesgue points in Sobolev type classes on metric measure spaces for $p > 0$

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Let (X, d, μ) be a metric space with the metric d and regular Borel measure μ satisfying the γ-doubling condition: for some constant $a_\mu > 0$ the following inequality is true $\mu(B(x,R)) \le a_\mu (R/r)^\gamma \mu(B(x,r))$, $0 < r < R$. Here $B(x,r) = \{y \in X : d(x,y) < r\}$ denotes the ball with the center at $x \in X$ and the radius $r > 0$.

If f is a measurable function on X and $\alpha > 0$, then $D_\alpha[f]$ denotes the class of all measurable functions g with the following property: there exist a subset $E \subset X$, $\mu(E) = 0$ such that for any $x, y \in X \setminus E$

$$|f(x) - f(y)| \le d^\alpha(x,y)[g(x) + g(y)].$$

For $\alpha, p > 0$ denote $M^p_\alpha(X) = \{f \in L^p(X) : L^p(X) \cap D_\alpha[f] \ne \varnothing\}$. These classes generate capacities $\mathrm{Cap}_{\alpha,p}$ in a natural way. We denote the s-Hausdorff content of E by $\mathbb{H}^s_\infty(E)$.

Theorem 1. *Let $\alpha > 0$, $0 < p < \gamma/\alpha$ and $f \in M^p_\alpha(X)$. Then there exists a set $E \subset X$ such that for any $x \in X \setminus E$,*

$$\lim_{r \to +0} I^{(p)}_{B(x,r)} = f^*(x), \quad \lim_{r \to +0} \fint_{B(x,r)} |f - f^*(x)|^q \, d\mu = 0, \quad \frac{1}{q} = \frac{1}{p} - \frac{\alpha}{\gamma},$$

and the following estimates hold:

1) *if* $\alpha > 0$, *then* $\dim_{\mathrm{H}}(E) \le \gamma - \alpha p$,

2) *if* $0 < \alpha \le 1$, *then* $\mathrm{Cap}_{\alpha,p}(E) = 0$.

Theorem 2. *Let $\alpha > 0$, $0 < p < \gamma/\alpha$, $0 < \beta < \alpha$. Then for any function $f \in M^p_\alpha(X)$ there exists a set $E \subset X$ such that*

1) $\mathbb{H}^{\gamma-(\alpha-\beta)p}_\infty(E) = 0$, *in particular*, $\dim_{\mathbb{H}}(E) \le \gamma - (\alpha - \beta)p$,

2) *for all* $x \in X \setminus E$

$$\lim_{r \to +0} r^{-\beta} \left(\fint_{B(x,r)} |f - f^*(x)|^q \, d\mu \right)^{1/q} = 0, \quad \text{where} \quad \frac{1}{q} = \frac{1}{p} - \frac{\alpha}{\gamma}.$$

In the case $p \ge 1$ these results are mainly known (see [1, 2] for $p > 1$, [3] for $p = 1$, and references in these papers).

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Approximation by sums of shifts of one function

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The aim of the talk is to discuss different results of the following type.

Theorem. *Let $\mathbb{T} = [-\pi, \pi)$ and the 2π-periodic function $f(t) = \sum_{n\in\mathbb{Z}} c_n e^{int}$ from the real space $L_2(\mathbb{T})$ be such that $c_0 = 0$, $c_n \neq 0$ for all $n \in \mathbb{Z} \setminus \{0\}$ and*

$$\sum_{n\in\mathbb{Z}} |n||c_n|^2 < \infty. \qquad (*)$$

Then the sums

$$\sum_{k=1}^{N} f(t + a_k), \qquad a_k \in \mathbb{R}, \quad N = 1, 2, \dots,$$

are dense in the space $L_2^0(\mathbb{T}) = \{g \in L_2(\mathbb{T}) : \int_\mathbb{T} g(t)\, dt = 0\}$.

The condition (*) in this theorem cannot be replaced by $c_n = O(1/n)$: sums of shifts of the function

$$f(t) = I_{(-\pi,-\alpha)}(t) - I_{(\alpha,\pi)}(t), \qquad t \in \mathbb{T},$$

(I_A denotes the indicator function of the set A) assume only integer values and therefore are not dense in $L_2^0(\mathbb{T})$.

Nonlinear maps preserving the minimum and surjectivity moduli

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Let X and Y be infinite-dimensional complex Banach spaces, and let $\mathcal{B}(X)$ (resp. $\mathcal{B}(Y)$) denote the algebra of all bounded linear operators on X (resp. on Y). We describe surjective maps φ from $\mathcal{B}(X)$ to $\mathcal{B}(Y)$ satisfying

$$\mathrm{c}('(\mathrm{S}) \pm '(\mathrm{T})) = \mathrm{c}(\mathrm{S} \pm \mathrm{T})$$

for all $S,\ T \in \mathcal{B}(X)$, where $\mathrm{c}(\cdot)$ stands either for the minimum modulus, or the surjectivity modulus, or the maximum modulus. We also obtain analog results for the finite-dimensional case.

Determinantal Point Processes and Their Quasi-Symmetries

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The classical De Finetti Theorem (1937) states that an exchangeable collection of random variables is a mixture of Bernoulli sequences. Here exchangeability means invariance under the action of the infinite group of finite permutations. We consider the weaker notion of quasi-invariance under which applying a permutation results in multiplication of the measure by a function, the Radon-Nikodym derivative.

The main result of the talk is that determinantal point processes on Z induced by integrable kernels are indeed quasi-invariant under the action of the infinite symmetric group. The Radon-Nikodym derivative is found explicitly. A key example is the discrete sine-process of Borodin, Okounkov and Olshanski.

The main result has a continuous counterpart: namely, it is proved that determinantal point processes with integrable kernles on R, a class that includes processes arising in random matrix theory such as the sine-process, the process with the Bessel kernel or the Airy kernel, are quasi-invariant under the action of the group of diffeomorphisms with compact support.

Discrete Littlewood-Paley type representations and sampling numbers in mixed order Sobolev spaces

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We consider mixed order Sobolev spaces

$$S^r_pW(\mathbb{T}^d) := \Big\{ f \in L_p(\mathbb{T}^d) : \|f|S^r_pW(\mathbb{T}^d)\| := \Big\| \Big(\sum_{j \in \mathbb{N}^d_0} 2^{2r|j|_1} |\delta_j[f](\cdot)|^2 \Big)^{\frac{1}{2}} \Big\|_p < \infty \Big\}$$

where $1 < p < \infty$ and $r > \frac{1}{p}$. As usual $\delta_j[f]$ denotes that part of the Fourier series of f with frequencies in dyadic rectangles. We study a replacement of $\delta_j[f]$ by building blocks that use only discrete information of f (function evaluations). Such a replacement (in the sense of equivalent norms) can be achieved with the help of tensorized Faber bases where a continuous function f is decomposed into tensor products of dilated and translated hat functions. Another way are tensorized sampling kernels based on trigonometric polynomials. The obtained discrete characterizations are well suited for studying sampling issues in $S^r_pW(\mathbb{T}^d)$. We construct sampling algorithms taking values on sparse grids that allow for proving asymptotically optimal error bounds for the linear sampling widths

$$g_n(S^r_pW(\mathbb{T}^d), Y) := \inf_{\substack{(\xi_i)_{i=1}^n \subset \mathbb{T}^d \\ (\psi_i)_{i=1}^n \subset Y}} \sup_{\|f|S^r_pW(\mathbb{T}^d)\| \le 1} \Big\| f(\cdot) - \sum_{i=1}^n f(\xi_i)\psi_i(\cdot) \Big| Y \Big\|,$$

where the error is measured either in Lebesgue spaces $L_q(\mathbb{T}^d)$ with $1 < q \le \infty$ as well as in isotropic Sobolev spaces $W^\gamma_q(\mathbb{T}^d)$, $r > \gamma > 0$, $1 < p \le q < \infty$. We compare the results to known results on linear widths and obtain that sampling and linear widths are equal in order if p and q are on the same side of 2. On the other hand, if $p < 2 < q$ then sampling widths are worse in order than linear widths.

Christoffel functions on Jordan arcs and curves with power type weights

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The talk is based on a joint work with Vilmos Totik. In this talk we establish asymptotics for Christoffel functions with respect to measures supported on Jordan arcs and curves having power type weights. The Christoffel function for a Borel measure μ is defined as

$$\lambda_n(\mu, z_0) = \inf_{\deg(P_n) \leq n} \int \frac{|P_n(z)|^2}{|P_n(z_0)|^2} d\mu(z),$$

where the infimum is taken for all polynomials of degree at most n. Christoffel functions play an important role in approximation theory and orthogonal polynomials, moreover they have many applications, for example, in random matrix theory. Asymptotics are established for $\lambda_n(\mu, z_0)$ where μ is supported on a union of Jordan curves and arcs Γ with the additional assumption that in a neighbourhood of z_0 the measure behaves like a power type weight measure, i.e. $d\mu(z) = |z|^\alpha ds_\Gamma(z), \alpha > -1$, where s_Γ is the arc length measure for Γ. The asymptotics on the curve components and on the arc components of Γ are very different. In curve components the asymptotics was established via a sharpened form of Hilbert's lemniscate theorem and polynomial inverse images, while on arc components a discretization of the equilibrium measure with respect to the zeros of a Bessel function was used.

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Interpolation of noethericity and index invariance on the scale of anisotropic spaces

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In this paper specific questions on operators Neothericity (see [1]) and index invariance on the scale of spaces are studied. For linear bounded operators acting on Banach spaces necessary and sufficient condition for index invariance, sufficient condition for Neotherian property interpolation is obtained. The examples are constructed which show the essentiality of obtained conditions. These results are used in the study of semi-elliptic operators on the scale of anisotropic spaces. For semi-elliptic operators some of previous results connected with Neothericity can be found in the papers [2-3].

Let

$$P(x,D) = \sum_{(\alpha:\nu)\leq s} a_\alpha(x) D^\alpha,$$

where $\alpha,\nu \in Z_+^n, \nu \neq 0$, $(\alpha : \nu) = \frac{\alpha_1}{\nu_1} + ... + \frac{\alpha_n}{\nu_n}$, $s \in N$, $D^\alpha = D_1^{\alpha_1}...D_n^{\alpha_n}$, $D_k = i\frac{\partial}{\partial x_k}$, $x = (x_1, ..., x_n) \in R^n$, $n \geq 2$, $a_\alpha(x)$ are infinitely differentiable and bounded with all derivatives.

For $k \in R, \nu \in Z_+^n$, let $H_\nu^k(R^n)$ denote

$$H_\nu^k(R^n) \equiv \left\{ u \in S' : \|u\|_{k,\nu}(R^n) = \left(\int_{R^n} |\hat{u}(\xi)|^2 (1 + |\xi|_\nu)^{2k} d\xi \right)^{1/2} < \infty \right\},$$

where $|\xi|_\nu = (\sum_{i=1}^n |\xi_i|^{2\nu_i})^{1/2}$, S' is the space of generalized functions of slow growth, $\hat{u}$ is the Fourier transform of u.

Let $\Omega \subset R^n$ be some domain. Denote by $\dot{H}_\nu^k(\Omega)$ the completion of $C_0^\infty(\Omega)$ with the norm of $H_\nu^k(R^n)$.

In this work, sufficient condition for the invariance of the index and Noethericity preserving is proved for semi-elliptic operator at one point.

This condition is restriction on the coefficients of the principal part of the operator.

For uniformly semi-elliptic operator the following result is obtained:

Theorem. *Let* $P(x, D) : \dot{H}_v^{k+s}(\Omega) \to \dot{H}_v^k(\Omega)$ *be uniformly semi-elliptic operator. If the operator* $P(x, D) : \dot{H}_v^{k_1+s}(\Omega) \to \dot{H}_v^{k_1}(\Omega)$ *is Noetherian for some* k_1*, then* $P(x, D) : \dot{H}_v^{k+s}(\Omega) \to \dot{H}_v^k(\Omega)$ *is Noetherian for any* k*, and* $ind_k(P)$ *does not depend on* k*.*

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On solvability of a Dirichlet problem with the boundary function in L_2 for the second-order elliptic equation

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We consider a Dirichlet problem in a bounded domain $Q \subset R_n$ for a general second-order elliptic equation with the boundary function in L_2. In the author's previous papers necessary and sufficient conditions for the existence of an $(n-1)$-dimensionally continuous solution were obtained under some natural assumptions on the equation coefficients. Those assumptions are formulated in terms of an auxiliary operator equation in a special Hilbert space and are difficult to verify. In the present work we obtain necessary and sufficient conditions for the existence of a solution in terms of the original problem for a more narrow class of the right-hand sides. It is shown that if in addition the boundary function is required to belong to $W_2^{1/2}(\partial Q)$ then obtained conditions transform into conditions of solvability in $W_2^1(Q)$.

On Cesaro summability of Fourier series of continuous functions

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The sequence

$$\sigma_n^\alpha(f,x) = \frac{1}{A_n^\alpha}\sum_{k=0}^{n} A_{n-k}^{\alpha-1} S_k(f,x), \quad n = 0,1,...,$$

where

$$A_0^\alpha = 1, \quad A_k^\alpha = \frac{(\alpha+1)(\alpha+2)...(\alpha+k)}{k!}$$

and $S_k(f,x)$ are partial sums of trigonometric Fourier series of integrable function f, is called the (C,α) means or the $\alpha-$order Cesaro means of f.

In this talk we present the uniform convergence of negative order Cesaro means of Fourier series of continuous functions after correction of these functions on a set of small measure. We also give an example of continuous function, the (C,α) means of Fourier series of which diverge over any subsequence on the set of positive measure for any $\alpha \in \left(-\frac{1}{2}, 1\right)$.

Definition 1. *The density $\rho(S)$ of the set of positive integers S is the quantity*

$$\rho(S) = \limsup_{n\to\infty} \frac{s_n}{n},$$

where s_n is the number of elements of S not exceeding n.

The following theorems are true:

Theorem 1. *There exists a function $f_0 \in C_{[-\pi,\pi]}$, such that for an arbitrary increasing sequence of natural numbers $\{m_\nu\}_{\nu=1}^\infty$ the set of points x for which*

$$\limsup_{\nu\to\infty} \left|\sigma_{m_\nu}^\alpha(f_0,x)\right| = +\infty$$

has a positive measure for all α, satisfying the inequality $-1 < \alpha < -1/2$.

Theorem 2. *There exists a set S of natural numbers of density 1, such that for any positive number ε and for any measurable, almost everywhere finite on $[-\pi,\pi]$ function $f(x)$ there exists a function $g(x) \in C_{[-\pi,\pi]}$, such that $\mu\{x : f \neq g\} < \varepsilon$ and the Cesaro means $\sigma_m^\alpha(g,x)$ of trigonometric Fourier series of function $g(x)$ converge to it uniformly on $[-\pi,\pi]$ as $m \to \infty, m \in S$ for any $\alpha < 0$, $\alpha \neq -1, -2, ...$*

The General Theory of Random Processes in Non-Standard Case

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Usually, in all investigations concerning the Theory of Random Processes or their applications, also in Statistics of Random Processes, the standard, so-called usual conditions are assumed to be true. However, in many situations, and particularly, in applications, it happens that this conditions do not hold. For this general case strong martingales (Mertens [1]), A-martingales (Lenglart [2]) and optional (or O-) martingales (Galchuk [3]) have been introduced and stochastic calculi with respect such martingales and martingale random measures were developed.

Recently, some authors have started new investigations on the theory of Random Processes and, in particular, to its applications in Stochastic Finance, without the usual assumptions on a stochastic basis (Kühn and Stroh [4], Czichowsky and Shachermayer [5], Gasparyan [6]). We will talk about this development.

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On uniqueness of Franklin series

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Let $\{f_n(x)\}_{n=0}^{\infty}$ be the orthonormal in $L_2[0;1]$ Franklin system. During our presentation we will talk about the following theorems:

Theorem 1. *If the Franklin series*

$$\sum_{n=0}^{\infty} a_n f_n(x) \tag{1}$$

converges in measure to a bounded function $f(x)$ and if for every $x \in [0;1]$

$$\sup_N \left| \sum_{n=0}^{N} a_n f_n(x) \right| < \infty, \tag{2}$$

then the series (1) is the Fourier-Franklin series of f.

Theorem 2. *If the Franklin series* (1) *with coefficients*

$$a_n = o(\sqrt{n}) \tag{3}$$

converges in measure to a bounded function $f(x)$, and everywhere, except of a countable set E, satisfies the condition (2)*, then the series* (1) *is the Fourier-Franklin series of f.*

Let k be a fixed natural number. We consider also a multiple Franklin series

$$\sum_{\mathbf{m}\in\mathbb{N}_0^k} a_{\mathbf{m}} f_{\mathbf{m}}(\mathbf{x}), \tag{4}$$

where $\mathbf{m} = (m_1, ..., m_k) \in \mathbb{N}_0^k$ is a vector with nonnegative integer coordinates, $\mathbf{x} = (x_1, ..., x_k) \in [0;1]^k$ and $f_{\mathbf{m}}(\mathbf{x}) = f_{m_1}(x_1) \cdots f_{m_k}(x_k)$.

We say that (4) converges by rectangles at the point $\mathbf{x}$, if there exists the following limit

$$\lim_{\mathbf{M}\to+\infty} \sum_{\mathbf{m}\leq\mathbf{M}} a_{\mathbf{m}} f_{\mathbf{m}}(\mathbf{x}),$$

where $\mathbf{m} \leq \mathbf{M}$ means $m_j \leq M_j$, $j = 1, ..., k$, and $\mathbf{M} = (M_1, ..., M_k) \to +\infty$ means that $\min_j M_j \to +\infty$.

By $\sigma_\nu(\mathbf{x})$ we denote the square-shaped partial sums of series (4) with indices 2^ν, i.e.

$$\sigma_\nu(\mathbf{x}) = \sum_{\mathbf{m}:m_j\leq 2^\nu} a_{\mathbf{m}} f_{\mathbf{m}}(\mathbf{x}), \tag{5}$$

where $\mathbf{m} = (m_1, ..., m_k)$.

Theorem 3. *The series* (4) *is the Fourier-Franklin series of the function $f \in L([0;1]^k)$ if and only if $\sigma_\nu(x)$ converges in measure to f and satisfies*

$$\liminf_{\lambda\to+\infty} \left(\lambda \cdot \mu \left\{x \in [0;1]^k : \sup_\nu |\sigma_\nu(x)| > \lambda \right\}\right) = 0.$$

Summability of two-dimensional Fourier series

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It is proved a BMO-estimation for quadratic and rectangular partial sums of two-dimensional Fourier series from which it is derived an almost everywhere exponential summability of quadratic and rectangular partial sums of double Walsh-Fourier series.

Norlund strong logarithmic means of double Fourier series acting from the space $L \log L$ into the space $L_p, 0 < p < 1$ are studied. The maximal Orlicz space such that the Norlund strong logarithmic means of double Fourier series for the functions from this space converge in two-dimensional measure is found.

We also consider the triangular summability method for two-dimensional Walsh-Fourier series.

Weak Greedy Algorithm and the Multivariate Haar Basis

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We investigate convergence of weak thresholding greedy algorithms for the multivariate Haar basis for $L_1[0,1]^d$ $(d \geq 1)$. We prove convergence and uniform boundedness of the weak greedy approximants for all $f \in L_1[0,1]^d$. Also we characterize all quasi-greedy subsystems of the Multivariate Haar basis. Also we talk a little bit about near uncoditionality of Haar basis.

Some results are joint with S.J. Dilworth and D. Kutzarova.

On the unconditional and absolute convergence of Haar series in L^p

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It is known that there is no unconditional basis for $L[0,1]$. Consequently, there exists a function $g_0(x) \in L^1[0,1]$, which cannot be represented by a Haar series $\sum_{n=0}^{\infty} b_n h_n(x)$ which converges unconditionally, or, even, absolutely, in the metric of $L^1[0,1]$.

We prove that for every function $f(x) \in L^1[0,1]$ one can find a Haar series $\sum_{n=0}^{\infty} b_n h_n(x)$, which absolutely (consequently, also unconditionally) converges to $f(x)$ in the $L^p[0,1]$ metric for all $p \in (0,1)$ i.e.

$$\lim_{n\to\infty} \int_0^1 \left| \sum_{k=0}^{n} b_k h_k(x) - f(x) \right|^p dx = 0 \quad \text{and} \quad \int_0^1 \left(\sum_{k=0}^{\infty} |b_k h_k(x)| \right)^p dx < \infty.$$

It is not hard to see that the Fourier-Haar series of the function

$$f_0(x) = \sum_{n=0}^{\infty} c_n(f) h_n(x) = \sum_{k=1}^{\infty} \sum_{m=1}^{2^k} \frac{1}{k 2^{\frac{k}{2}}} h_{2^k+m}(x) \in \ L^2_{[0,1]},$$

is unconditionally convergent in $L^p[0,1]$, $p \in (0,2]$ (i.e. in the $L^p[0,1]$ metric), but

$$\lim_{N\to\infty} \int_0^1 (\sum_{n=0}^{N} |c_n(f) h_n(x)|)^p dx = \lim_{N\to\infty} (\sum_{k=1}^{N} \frac{1}{k})^p = \infty, \forall p \in (0,2].$$

Theorem 1. *For any $p \in (0,1)$ and every function $f(x) \in L^p[0,1]$ one can find a Haar series $\sum_{n=1}^{\infty} b_n h_n(x)$, which converges absolutely to $f(x)$ in the $L^p[0,1]$ metric, i.e.*

$$\lim_{n\to\infty} \int_0^1 \left| \sum_{k=0}^{n} b_k h_k(x) - f(x) \right|^p dx = 0, \quad \text{and} \quad \int_0^1 (\sum_{k=0}^{\infty} |b_k h_k(x)|)^p dx < \infty.$$

Theorem 2. *For every $\varepsilon > 0$, there exists a measurable set $E \subset [0,1]$ with $|E| > 1 - \varepsilon$, such that for every function $f(x) \in L[0,1]$ one can find a function $\tilde{f}(x) \in L[0,1]$, $\tilde{f}(x) = f(x)$, $x \in E$, which Fourier series with respect to the Haar system converges absolutely to $\tilde{f}(x)$ in the $L^p[0,1]$ metric for all $p \in (0,1)$ and $\{c_k(f) = \int_0^1 \tilde{f}(x)h_k(t)dt, \forall k \in spec(\tilde{f})\} \searrow 0$.*

The spectrum of $f(x)$ (denoted by $spec(f)$) is the support of $c_k(f)$, i.e. the set of integers for which $c_k(f)$ is non-zero.

Note that these theorems are not true for the trigonometric system, i.e. the trigonometric system is not an absolute representation system for the space $L^p[0,1]$, $\forall p \in (0,1)$.

At the end we list some open questions:

Question 1. Is the trigonometric system a representation system for the unconditional convergence for the space $L^p[0,1]$ for some $p \in (0,1)$?

Question 2. Are Theorems 1,2 true for Franklin system?

Question 3. Is the Theorem 2 true in the case $p = 1$?

On multivariate segmental interpolation problem

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The following problem is considered in our talk, which we call segmental interpolation problem, or, briefly, segmental problem: Suppose $\mathcal{X}_I = \{\mathbf{x}^{(\nu)} : \nu \in I\}$ is a finite or infinite set of knots in $\mathbb{R}^d$. Suppose also that $\mathcal{S}_I = \{[\alpha_\nu, \beta_\nu] : \nu \in I\}$ is a set of arbitrary segments. The segmental problem $\{\mathcal{X}, S\}_I^n$ is to find a polynomial p in d variables and of total degree less than or equal to n, satisfying the conditions

$$\alpha_\nu \leq p(\mathbf{x}^{(\nu)}) \leq \beta_\nu, \ \ \forall \nu \in I.$$

We give a necessary and sufficient condition for the solvability of the segmental problem (see [1]). In the case when the problem is solvable and the set of knots $\mathcal{X}_I$ is finite, we give a method to find a solution of the segmental problem.

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About an approach in spectral theory

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Let $\mu_n(q,\alpha,\beta)$, $n=0,1,2,\dots$, are the eigenvalues of the Sturm-Liouville problem $L(q,\alpha,\beta)$:

$$-y''+q(x)\,y=\mu y,\ x\in(0,\pi),\ q\in L^1_R[0,\pi],$$

$$y(0)\cos\alpha+y'(0)\sin\alpha=0,\ \alpha\in(0,\pi],$$

$$y(\pi)\cos\beta+y'(\pi)\sin\beta=0,\ \beta\in[0,\pi).$$

The first question that we want to answer is:

How the eigenvalues of the problem are moving, when (α,β) runs on $(0,\pi]\times[0,\pi)$.

For this purpose we introduce the concept of the eigenvalues function (EVF).

Definition: *The function $\mu_q(\cdot,\cdot)$, defined on $(0,\infty)\times(-\infty,\pi)$ by the formula*

$$\mu_q(\alpha+\pi k,\beta-\pi m)\overset{def}{=}\mu_{k+m}(q,\alpha,\beta),\ k,m=0,1,2,\dots,$$

is called the eigenvalues function (EVF) of the family of problems $\{L(q,\alpha,\beta), \alpha \in (0,\pi], \beta \in [0,\pi)\}$.

We study some properties of this function, and, in the result, the answer to our first question is:

When (α, β) runs over $(0,\pi] \times [0,\pi)$, then the set of eigenvalues form an analytic surface, and we call that surface EVF.

We find necessary and sufficient conditions for a function of two variables having these properties to be the EVF of the family of problems $\{L(q,\alpha,\beta), \alpha \in (0,\pi], \beta \in [0,\pi)\}$. In particular, an algorithm for solving the inverse problem is given.

Riemann boundary problem in weighted spaces

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Let $\rho(t) = |t-t_1|^{\alpha_1}...|t-t_m|^{\alpha_m}, t_k \in T$, where $T = \{t, |t| = 1\}$ is the unit circle, and $\alpha_k, k = 1,2,...,m$, are real numbers. We denote

$$\rho_r(t) = \rho^*(t)|r^{\delta_1}t - t_1|^{n_1}...|r^{\delta_m}t - t_m|^{n_m}$$

and $\rho^*(t) = |t-t_1|^{\lambda_m}...|t-t_1|^{\lambda_m}$, where

$$\delta_k = \begin{cases} 1, & \text{if } \alpha_k \le -1\ , \\ 0, & \text{if } \alpha_k > -1\ , \end{cases}$$

$$n_k = \begin{cases} [\alpha_k] + 1, & \text{if } \alpha_k \text{ isn't an integer,} \\ \alpha_k, & \text{if } \alpha_k \text{ is an integer} \end{cases}$$

and $\lambda_k = \alpha_k - n_k$. It's clear that $\lambda_k \in (-1, 0]$ and $\rho^*(t) \in L^1(T)$.

We consider the problem R in the following statement:

Problem R. Let f be an arbitrary function in T belonging to the class $L^1(\rho)$. Find an analytic function $\Phi(z)$, $\Phi(\infty) = 0$ in $D^+ \cup D^-$, where $D^+ = \{z; |z| < 1\}$, $D^- = \{z; |z| > 1\}$ such that

$$\lim_{r \to 1-0} \|\Phi^+(rt) - a(t)\Phi^-(r^{-1}t) - f(t)\|_{L^1(\rho_r)} = 0, \tag{1}$$

where $a(t), a(t) \neq 0$ is an arbitrary function from $C^\delta(T)$,$\delta > 0$, $\Phi^\pm$ are restrictions of Φ on $D^\pm$ respectively. Let $\kappa = ind\big(a(t)\big), t \in T$. The analogous problem with $\rho(t) \equiv 1$ is investigated in [1].

In this work we prove that if $\sum\limits_{k=1}^{m} n_k + \kappa \geq 0$, then the problem R is solvable for any function f from $L^1(\rho)$. In case of $\sum\limits_{k=1}^{m} n_k + \kappa < 0$ we give necessary and sufficient conditions for the solvability of this problem. Besides, solutions are given in explicit form.

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Asymptotic behaviour of Besov norms via wavelet type basic expansions

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J. Bourgain, H. Brezis, P. Mironescu [1] proved the following asymptotic formula: if $\Omega \subset R^d$ is a bounded domain with smooth boundary, then

$$\lim_{s \nearrow 1}(1-s)\int_\Omega\int_\Omega \frac{|f(x)-f(y)|^p}{|x-y|^{d+sp}}dxdy = K\int_\Omega |\nabla f(x)|^p dx,$$

where $f \in W^{1,p}(\Omega)$, $1 < p < \infty$ and K is a constant depending only on p and d.

This result has attracted a lot of interest, and it has been generalized and extended by several authors. There are versions of the above formula for Besov norms and for real interpolation spaces. Let me mention papers by V. Maz'ya and T. Shaposhnikova (2002), M. Milman (2005), G.E. Karadzhov, M. Milman and J. Xiao (2005), H. Triebel (2011), R. Arcangéli and J.J. Torrens (2013).

The purpose of this talk is to present analogous asymptotic formulae for some norms in Besov spaces, which are defined using coefficients of the basic expansion of a function with respect to a wavelet or a wavelet type basis. We cover both the case of usual (isotropic) Besov and Sobolev spaces, as well as Besov and Sobolev spaces with dominating mixed smoothness. We treat also the Besov type spaces defined in terms of Ditzian – Totik modulus of smoothness, but for a restricted range of parameters only.

The results presented in the talk are contained in [2].

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On some exponential estimates of Hilbert transform and strong convergence in measure of multiple Fourier series

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Let $\mathbb{T}^d$ be the d-dimensional torus and $f(\mathbf{x}) = f(x_1, \ldots, x_d) \in L^1(\mathbb{T}^d)$ be an arbitrary Lebesgue integrable function with the multiple Fourier series

$$\sum_{(k_1,\ldots,k_d)\in\mathbb{Z}^d} a_{\mathbf{k}} e^{i(k_1x_1+\ldots+k_dx_d)}.$$

Denote the rectangular and the cubical partial sums of this series by

$$S_{\mathbf{n}}(\mathbf{x}, f) = S_{n_1,\ldots,n_d}(\mathbf{x}, f) = \sum_{\mathbf{k}\in\mathbb{Z}^d:\, 1\le|k_i|\le n_i} a_{\mathbf{k}} e^{i\mathbf{kx}}, \quad \mathbf{n} \in \mathbb{N}^d$$

$$S_n(\mathbf{x}, f) = \sum_{\mathbf{k}\in\mathbb{Z}^d:\, 1\le k_i\le n} a_{\mathbf{k}} e^{i\mathbf{kx}}, \quad n \in \mathbb{N},$$

respectively. We prove the following theorems.

Theorem 1. *If $\varepsilon > 0$ and $f \in L(\log L)^{d-1}(\mathbb{T}^d)$, then there exists a set $E \subset \mathbb{T}^d$*

such that $|E| > |\mathbb{T}^d| - \varepsilon$ *and*

$$\int_{\mathbb{T}^d} \exp\left(c_1\varepsilon\frac{|S_{n_1,\ldots,n_d}(\mathbf{x},f)|}{\|f\|_{L(\log L)^{d-1}(\mathbb{T}^d)}}\right)^{1/d} d\mathbf{x} \le c_2, \quad \mathbf{n}\in\mathbb{N}^d, \tag{1}$$

where c_1 *and* c_2 *are absolute constants.*

Theorem 2. *If* $\varepsilon > 0$ *and* $f \in L(\log L)^{d-1}(\mathbb{T}^d)$, *then there exists a set* $E \subset \mathbb{T}^d$ *such that* $|E| > |\mathbb{T}^d| - \varepsilon$ *and*

$$\int_{\mathbb{T}^d} \exp\left(c_1\varepsilon\frac{|S_n(\mathbf{x},f)|}{\|f\|_{L(\log L)^{d-1}(\mathbb{T}^d)}}\right) d\mathbf{x} \le c_2, \quad n\in\mathbb{N}, \tag{2}$$

where c_1 *and* c_2 *are absolute constants.*

Applying these theorems, we deduce a property of strong convergence in measure and some estimates of the growth for the partial sums of multiple Fourier series. One and two dimensional cases of Theorem 1 was considered in the papers [2, 3]. The papers [1, 4] prove that the class $L\log^{d-1}L(\mathbb{T}^d)$ is optimal in such estimates. We deduce these theorems establishing some exponential estimates of the multiple Hilbert transform and sparse operators recently introduced in [5]. We consider analogous problems for the multiple Walsh and rearranged Haar series.

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On an equivalency of differentiation basis of dyadic rectangles

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The paper considers differentiation properties of rare basis of dyadic rectangles corresponding to an increasing sequence of integers $\{\nu_k\}$. We prove that the condition

$$\sup_k(\nu_{k+1} - \nu_k) < \infty$$

is necessary and sufficient for such basis to be equivalent to the full basis of dyadic rectangles.

On a new family of weighted integral representations of holomorphic functions in the unit ball of $\mathbf{C}^n$

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Denote by B_n the unit ball in the complex n-dimensional space C^n : $B_n = \{w \in C^n : |w| < 1\}$. For $1 \leq p < +\infty$ and $\alpha > -1$ denote by $H^p_\alpha(B_n)$

the space of all functions f holomorphic in B_n and satisfying the condition

$$\int_{B_n} |f(w)|^p (1-|w|^2)^\alpha dm(w) < +\infty, \tag{1}$$

where dm is the Lebesgue measure in $C^n \equiv R^{2n}$. Further, for a complex number β with $Re\beta > -1$ denote

$$c_n(\beta) = \frac{\Gamma(n+1+\beta)}{\pi^n \Gamma(1+\beta)}. \tag{2}$$

The following theorem is valid (W.Wirtinger (1932), M.M.Djrbashian (1945,1948), L.K.Hua (1958), F.Forelli and W.Rudin (1974), M.M.Djrbashian (1987,1988)):

Theorem 1. *Assume that $1 \le p < +\infty, \alpha > -1$ and a complex number β satisfy the condition*

$$Re\beta \ge \alpha, \quad p = 1,$$
$$Re\beta > \frac{\alpha+1}{p} - 1, \quad 1 < p < \infty. \tag{3}$$

Then any function $f \in H^p_\alpha(B_n)$ admits the following integral representations:

$$f(z) = c_n(\beta) \int_{B_n} \frac{f(w)(1-|w|^2)^\beta}{(1- < z, w >)^{n+1+\beta}} dm(w), \quad z \in B_n, \tag{4}$$

$$\overline{f(0)} = c_n(\beta) \int_{B_n} \frac{\overline{f(w)}(1-|w|^2)^\beta}{(1- < z, w >)^{n+1+\beta}} dm(w), \quad z \in B_n, \tag{5}$$

where $< \cdot, \cdot >$ is the Hermitean inner product in C^n.

In the present report a family of kernels reproducing the functions from $H^p_\alpha(B_n)$ are constructed.

Theorem 2. *Assume that $1 \le p < +\infty, \alpha > -1$ and a complex number β satisfy the condition (3). Moreover, assume that $\rho > 0, Re\gamma > -n$ and $\mu = \frac{\gamma+n}{\rho}$. Then any function $f \in H^p_\alpha(B_n)$ admits the following integral representations:*

$$f(z) = \int_{B_n} f(w) \cdot S_{\beta,\rho,\gamma}(z;w)(1-|w|^{2\rho})^\beta |w|^{2\gamma} dm(w), \quad z \in B_n, \tag{6}$$

$$\overline{f(0)} = \int_{B_n} \overline{f(w)} \cdot S_{\beta,\rho,\gamma}(z;w)(1-|w|^{2\rho})^{\beta}|w|^{2\gamma}dm(w), \quad z \in B_n, \qquad (7)$$

where the kernel $S_{\beta,\rho,\gamma}(z;w)$ has the following properties :

(a) $S_{\beta,\rho,\gamma}(z;w)$ *is holomorphic in* $z \in B_n$ *;*

(b) $S_{\beta,\rho,\gamma}(z;w)$ *is antiholomorphic in* $w \in B_n$ *and continuous in* $w \in \overline{B_n}$*;*

(c) *for* $z \in B_n, w \in \overline{B_n}$

$$|S_{\beta,\rho,\gamma}(z;w)| \leq const(n;\beta;\rho;\gamma)(1-|z|)^{-(n+1+Re\beta)}; \qquad (8)$$

(d) *for* $z \in B_n, w \in \overline{B_n}$

$$S_{\beta,\rho,\gamma}(z;w) = \frac{\rho}{\pi^n\Gamma(\beta+1)} \cdot \int_0^{+\infty} e^{-t} \cdot t^{\mu+\beta} \cdot E_\rho^{(n)}(t^{1/\rho} < z,w >;\mu)dt. \quad (9)$$

Here

$$E_\rho^{(n)}(\eta;\mu) \equiv \sum_{k=0}^{\infty} \frac{\Gamma(k+n)}{\Gamma(k+1)} \cdot \frac{\eta^k}{\Gamma(\mu+\frac{k}{\rho})}, \qquad \eta \in C, \qquad (10)$$

is the Mittag-Leffler type entire function.

Integral representation through the differentiation operator and embedding theorems for multianisotropic spaces

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Let R^n be the n-dimensional space, Z_+^n be the set of multi-indices. For the set of multi-indices we denote by $\aleph$ the smallest convex polyhedron containing all the points of that set. The polyhedron is said to be completely correct, if:

a) it has a vertex in the origin of coordinates and in all coordinate axes;

b) outward normals of all $(n-1)$-dimensional non-coordinate faces are positive.

Let μ_i be the outward normal of the face $\aleph_i^{(n-1)}$ such that $\forall\alpha \in \aleph_i^{(n-1)}$, $(\alpha, \mu^i) = \alpha_1\mu_1^i + \cdots + \alpha_n\mu_n^i = 1$; $|\mu^i| = |\mu_1^i| + \cdots + |\mu_n^i|$. Let us denote by $W_p^{\aleph}(R^n)$ the set of all measurable functions in R^n for which $f \in L_p(R^n)$ and for any $\forall\alpha^i \in \aleph_i^{(n-1)}$, $D^{\alpha_i}f \in L_p(R^n), i = 1, \cdots, M$.

In the present work an integral representation through the differentiation operator is offered, which is generated via the polyhedron $\aleph$, and, applying the obtained integral representation, embedding of the set $W_p^{\aleph}(R^n)$ in $L_q(R^n)$ is proved.

Theorem. *Let $\aleph$ be a convex polyhedron and $f \in L_p(R^n)$ and $\forall\alpha \in \partial'\aleph$, $D^{\alpha_1}f \in L_p(R^n)$. Let multi-index β and numbers $1 \leq p \leq q \leq \infty$ be such that $(\beta;\mu) + \left(\frac{1}{p} - \frac{1}{q}\right)|\mu| < 1$, for any normal μ of the $(n-1)$ -dimensional hyper-plane of the polyhedron $\aleph$.*

Let

$$\max(\beta;\mu) + \left(\frac{1}{p} - \frac{1}{q}\right)|\mu| = (\beta;\mu_0) + \left(\frac{1}{p} - \frac{1}{q}\right)|\mu_0|.$$

Then $D^\beta W_p^{\aleph}(R^n)$ is embedded $L_q(R^n)$, i.e. for any $f \in W_p^{\aleph}(R^n)$, the derivative $D^\beta f \in L_q(R^n)$ exists, and the following estimate is true:

$$\|D^\alpha f\|_{L_q(R^n)} \leq C_1 h^{1-\left((\beta;\mu_0)+\left(\frac{1}{p}-\frac{1}{q}\right)|\mu_0|\right)} \sum_{i=1}^{M} \|D^\alpha f\|_{L_p(R^n)}$$

$$+C_2 h^{-\left((\beta;\mu_0)+\left(\frac{1}{p}-\frac{1}{q}\right)|\mu_0|\right)} \|f\|_{L_p(R^n)},$$

where C_1, C_2 are numbers independent of f, h , and h is a parameter, which varies in $0 < h < h_0$.

Mixed norm variable exponent Bergman space on the unit disc

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This is a joint work with S. Samko.

We introduce and study the mixed norm variable order Bergman space $\mathcal{A}^{q,p(\cdot)}(\mathbb{D})$, $1 \leqslant q < \infty$, $1 \leqslant p(r) \leqslant \infty$, on the unit disk $\mathbb{D}$ in the complex plane. The mixed norm variable order Lebesgue - type space $\mathcal{L}^{q,p(\cdot)}(\mathbb{D})$ is defined by the requirement that the sequence of variable exponent $L^{p(\cdot)}(I)$ - norms of the Fourier coefficients of the function f belongs to l^q. Then $\mathcal{A}^{q,p(\cdot)}(\mathbb{D})$ is defined to be the subspace of $\mathcal{L}^{q,p(\cdot)}(\mathbb{D})$ which consists of analytic functions. We prove the boundedness of the Bergman projection and reveal the dependence of the nature of such spaces on possible growth of variable exponent $p(r)$ when $r \to 1$ from inside the interval $I = (0,1)$. The situation is quite different in the cases $p(1) < \infty$ and $p(1) = \infty$. In the case $p(1) < \infty$ we also characterize the introduced Bergman space $\mathcal{A}^{2,p(\cdot)}(\mathbb{D})$ as the space of Flett's fractional derivatives of functions from the Hardy space $H^2(\mathbb{D})$. The case $p(1) = \infty$ is specially studied, and an open problem is formulated in this case. We also reveal a condition on the growth from below of $p(r)$ when $r \to 1$, under which $\mathcal{A}^{2,p(\cdot)}(\mathbb{D}) = H^2(\mathbb{D})$ up to norm equivalence, and also find a condition on the growth from above of $p(r)$ when this is not longer true.

Searching for big submatrices with small norms in a given matrix

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During our talk we will discuss some results on norm estimates above of some big submatrices of a given matrix, defining a unit norm operator from $l^2(n)$ to $l^1(N)$.

Compactness criterion in the spaces of measurable functions

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Let (X, d, μ) be a bounded metric space with the metric d and regular Borel measure μ satisfying the doubling condition: for some constant $a_\mu > 0$ the following inequality is true:

$$\mu(B(x, 2r)) \leq a_\mu \mu(B(x, r)), \quad x \in X, \quad r > 0,$$

where $B(x, r) = \{y \in X : d(x, y) < r\}$ is the ball of the radius $r > 0$ centered at $x \in X$. Let $L^0(X)$ be the set of all (equivalence classes of) measurable functions on X. It is a complete metric space with respect to the metric

$$d_{L^0}(f, g) = \int_X \varphi_0(f - g)\, d\mu, \qquad \varphi_0(t) = \frac{|t|}{1 + |t|}.$$

The convergence in $L^0(X)$ coincides with the convergence in measure.

Let Ω be the class of increasing functions $\eta : (0, 1] \to (0, +\infty)$, such that $\eta(+0) = 0$, and Φ be the set of all even functions $\varphi : \mathbb{R} \to \mathbb{R}$, positive and increasing on $(0, +\infty)$, such that

$$\varphi(0) = \varphi(+0) = 0, \quad \lim_{t \to +\infty} \varphi(t) = \infty.$$

Consider the following maximal function

$$\mathcal{A}_\eta^\varphi f(x) = \sup_{B \ni x} \frac{1}{\eta(r_B)\mu(B)} \inf_{c \in \mathbb{R}} \int_B \varphi(f - c)\, d\mu, \quad 0 < t < 1,$$

where sup is taken over all balls containing the point x, r_B is the radius of B.

Theorem. *The set $S \subset L^0(X)$ is completely bounded if and only if the following condition holds:*

$$\lim_{\lambda \to +\infty} \sup_{f \in S} \mu\{|f| > \lambda\} = 0,$$

and there exist functions $\eta \in \Omega$ and $\varphi \in \Phi$ such that

$$\lim_{\lambda \to +\infty} \sup_{f \in S} \mu\{\mathcal{A}_\eta^\varphi f > \lambda\} = 0.$$

A similar compactness criterion was proved in [1] with another maximal function instead of $\mathcal{A}$ (where $f(x)$ stays on the place of the constant of the best approximation). Our criterion is stronger in sufficiency part.

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Elementary proof of the Meyer's Theorem of the equivalence of the sets of trigonometric polynomials

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In this talk we will study the existence of the isomorphism between the spaces $L^1_E(\mathbb{T})$ and $L^1_F(\mathbb{T}^\infty)$, where by $L^p_A(\mathbb{T}^k)$ we denote the following subspace of the Banach space $L^p(\mathbb{T}^k)$:

$$L^p_A(\mathbb{T}^k) := \{f \in L^p(\mathbb{T}^k) : \operatorname{supp} \widehat{f} \subset A\}.$$

Let $(a_n)_{n\in\mathbb{N}}$ be a fixed lacunary sequence of natural numbers. We define sets $E \subset \mathbb{Z}$ and $F \subset \mathbb{Z}^{\mathbb{N}}$, where by $\mathbb{Z}^{\mathbb{N}}$ stands for the dual group to $\mathbb{T}^{\mathbb{N}}$, in the following way:

$$F := \{\boldsymbol{\lambda} \in \mathbb{Z}^{\mathbb{N}} \ : \ |\lambda_n| \leq 1\}, \tag{1a}$$

$$E := \{\beta \in \mathbb{Z} \ : \ \beta = \sum_{k=1} a_k \lambda_k \text{ for } \lambda_n \in F\}. \tag{1b}$$

In his paper Y. Meyer [1] proved that whenever a sequence $(a_n)_{n\in\mathbb{N}}$ satisfies the condition $\sum_{j=1}^{\infty} \frac{|a_j|}{|a_{j+1}|} < \infty$, then the operator $T : L^\infty_F(\mathbb{T}^{\mathbb{N}}) \to L^\infty_E(\mathbb{T})$, given by the formula

$$Tf(x) = \sum_{\boldsymbol{\lambda}\in F} \widehat{f}(\boldsymbol{\lambda}) e^{2\pi i \langle \boldsymbol{\lambda}, \tau\rangle x},$$

is an isomorphism. However, Y. Meyer's argument seems to be incomplete in the case of the spaces $L^1_E(\mathbb{T})$ and $L^1_F(\mathbb{T}^\infty)$. In the case of L^1 norm we will give an elementary proof. The talk is based on a joint work with M. Wojciechowski.

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Real Interpolation in variable exponent Lebesgue spaces

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With upcoming applications in stochastics, fluid dynamics and image processing, Lebesgue spaces $L_{p(\cdot)}(\mathbb{R}^n)$ with variable integrability attract more and more attention nowadays. In this talk we give a negative answer to a question posed in [1], if Marcinkiewicz interpolation does hold on these variable exponent function spaces. Parallel we define and investigate Lorentz spaces with variable exponents and show that they can be obtained by real interpolation of $L_{p(\cdot)}((R))$ with $L_\infty(\mathbb{R})$. This talk is based on a joint work [2] with Jan Vybíral from Prague.

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Unconditionality of Franklin system with zero mean in $H^1(R)$

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In this talk we will give a necessary and sufficient condition for the grid point sequence for which the corresponding Franklin system with zero mean, being an orthonormal system of piecewise linear functions, is an unconditional basis in $H^1(R)$.

On solvability of initial-boundary problems for quasilinear parabolic systems in weighted Holder spaces

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We consider the following problem:

$$\begin{aligned}
&\frac{\partial u}{\partial t}+\mathcal{A}\left(x,t,\frac{\partial}{\partial x}\right)u=f(x,t,u,...,\nabla_{2b}u), \qquad x\in\Omega\subset\mathbb{R}^n\\
&\mathcal{B}\left(x,t,\tfrac{\partial}{\partial x}\right)u\Big|_{\partial\Omega}=0\\
&u\Big|_{t=0}=\varphi(x),
\end{aligned} \tag{1}$$

where $u=(u_1,u_2,...,u_n)$, $\nabla_q u$ is the vector with the components of all possible derivatives $D_x^{\beta}u_i$ of order $|\beta|=q$, $\mathcal{A}$ is a matrix differential operator of the order $2b$ with the elements A_{kj}, $\mathcal{B}$ is a matrix differential operator with the elements B_{ij} of order σ_i+2b, $\sigma_i\le -1$. We assume that the operator $\mathcal{L}u=\dfrac{\partial u}{\partial t}+\mathcal{A}u$ is parabolic in the I. Petrovskii's sense, $\mathcal{B}$ is in complementary relation with $\mathcal{L}$, and the function f is of the form

$$f(x,t,u,...,\nabla_{2b}u)=$$

$$=\sum_{|\alpha|\le 2b-k}D_x^{\alpha}\left(\sum_{k\le|\beta|\le 2b-|\alpha|}f_{\alpha\beta}(x,t,u,...,\nabla_k u)D^{\beta}u+f_{\alpha}(x,t,u,...,\nabla_k u)\right),$$

where $0\le k<2b$. We have obtained the following result:

Theorem. *Assume $\partial\Omega \in C^{2b+\alpha}$. Then there exist $M(T)$ and $\delta(T)$ such that if $\varphi \in C^s(\Omega)$, $s \in [k, 2b)$ and if the following consistency conditions are satisfied:*

$$\sum_{j=1}^{m} B_{qj}\left(x, 0, \frac{\partial}{\partial x}\right) \varphi_j \Big|_{\partial\Omega} = 0, \qquad \forall q: \quad \sigma_q + 2b \leq s,$$

and if $|\varphi|_{\Omega}^{(s)} \leq \delta(T)$, then the problem (1) has a unique solution u with

$$u \in C_s^{2b+\alpha, 1+\frac{\alpha}{2b}}(Q_T) \qquad \text{and} \qquad |u|_{s,Q_T}^{(2b+\alpha)} \leq M(T)|\varphi|_{\Omega}^{(s)}.$$

The implicit function theorem for a system of inequalities

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Consider strictly differentiable functions $f_i(x, y), i \in [1:k]$ on the space R^{n+m}, where $x \in R^n, y \in R^m$.

Suppose $(x_0, y_0) \in R^{n+m}$ is a point, and $f_i(x_0, y_0) \leq 0, i \in [1:k]$.

Denote

$$I(x_0, y_0) = \{i \in [1:k] : f_i(x_0, y_0) = 0\}.$$

Set

$$K = \{(\overline{x}, \overline{y}) \in R^{n+m} : (\frac{\partial f_i(x_0, y_0)}{\partial x}, \overline{x}) + (\frac{\partial f_i(x_0, y_0)}{\partial y}, \overline{y}) \leq 0, i \in [1:k]\},$$

$$D(\overline{x}) = \{\overline{y} \in R^m : (\overline{x}, \overline{y}) \in K\}.$$

Theorem. *Assume that the above mentioned conditions hold, and there exists a vector $w \in R^m$ such that*

$$(\frac{\partial f_i(x_0, y_0)}{\partial y}, w) < 0, \quad i \in I(x_0, y_0).$$

Then for any vector $(\hat{x}, \hat{y}) \in K$ *there exists a continuous mapping* $y : R^n \to R^m$ *defined in some neighbourhood* V *of the point* x_0 *such that*
a) $f_i(x, y(x)) \leq 0, i \in [1 : k], x \in V, y(x_0) = y_0$,
b) the derivative of the mapping $y'(x_0, \overline{x})$ *at* x_0 *exists in any direction* $\overline{x} \in R^n$ *and* $y'(x_0, \hat{x}) = \hat{y}, y'(x_0, \overline{x}) \in D(\overline{x}), \overline{x} \in R^n$.

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The Reconstruction Property in Banach Spaces Generated by Matrices

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The reconstruction property for Banach spaces was introduced by Casazza and Christensen. In this paper we give a type of the reconstruction property in Banach spaces which is generated by the Toeplitz matrices, and we call it the Toeplitz reconstruction property. It is proved that the standard reconstruction property in a Banach space can generate the Toeplitz reconstruction property from a given Toeplitz matrix but not conversely. Sufficient conditions on infinite matrices to have the reconstruction property for a discrete signal space are given.

Some property of Fourier-Franklin series

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We will discuss a.e. absolute (in L^p norm) convergence of Fourier-Franklin series.

Definition. We say that the basis $\{\varphi_n(x)\}_{n=1}^{\infty}$ of $C[0,1]$ has the property (D), if for any measurable set $E \subset [0,1], |E| > 0$ and condensation point x_0, there exists a continuous function $f_0(x)$ such that the Fourier series of any bounded function $g(x)$ coinciding with f_0 on E, absolutely diverges at the point x_0.

Theorem 1. *The Haar system has the property* (D).

Theorem 2. *The Franklin system has the property* (D).

Note that any subsystem of Haar system, which contains infinitely many packets, has the property (D).

From the paper [1] it follows that the Faber-Schauder system do not possess the property (D).

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On the distribution of interpolation points of multipoint Pade approximants with unbounded degrees of the denominators

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Given a regular compact set E in $\mathbb{C}$, a unit measure μ supported by ∂E, a triangular point set $\beta := \{\{\beta_{n,k}\}_{k=1}^{n}\}_{n=1}^{\infty}, \beta \subset \partial E$ and a function f, holomorphic on E and not meromorphic in $\mathbb{C}$, let $\pi_{n,m_n}^{\beta,f}$ be the associated multipoint β-Padé approximant of order (n, m_n), where $m_n = o(n)$, $n \to \infty$. Our main result is that if the function f has a multivalued singularity on the boundary of the domain of meromorphy and if the sequence $\pi_{n,m_n}^{\beta,f}$ converges exactly maximally to f relatively to the measure μ as $n \in \Lambda \subset \mathbb{N}$, then the points $\beta_{n,k}$ are uniformly distributed on ∂E with respect to μ as $n \in \Lambda$.

H-symmetric MRA-based wavelet frames

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A symmetry is one the most desirable properties for wavelet systems in applications. For an arbitrary symmetry group H, we give explicit formulas for refinable masks that are H-symmetric and have sum rule of order n. The description of all such masks is given. Several methods for the construction of H-symmetric wavelets (and multi-wavelets) providing approximation order n in different setups are developed.

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Luzin approximation for Sobolev type classes on metric measure spaces for $p > 0$

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Let (X, d, μ) be a metric space with the metric d and regular Borel measure μ satisfying the γ-doubling condition: for some constant $a_\mu > 0$ the following inequality is true $\mu(B(x,R)) \le a_\mu (R/r)^\gamma \mu(B(x,r))$, $0 < r < R$. Here $B(x,r) = \{y \in X : d(x,y) < r\}$ denotes the ball of the radius $r > 0$ with the center at $x \in X$.

If f is a measurable function on X and $\alpha > 0$, then $D_\alpha[f]$ denotes the class of all measurable functions g with the following property: there exists a subset $E \subset X$, $\mu(E) = 0$, such that for all $x, y \in X \setminus E$

$$|f(x) - f(y)| \le d^\alpha(x,y)[g(x) + g(y)].$$

For $\alpha, p > 0$ we denote

$$M_\alpha^p(X) = \{f \in L^p(X) : L^p(X) \cap D_\alpha[f] \ne \varnothing\}.$$

These (Hajłasz–Sobolev) classes generates the capacities $\mathrm{Cap}_{\alpha,p}$ in a natural way.

The Hausdorff content of E is denoted by $\mathbb{H}_\infty^s(E)$, $H^\beta(X)$ stands for the usual Hölder classes on X of the degree $\beta > 0$.

Theorem. *Let $0 < \beta \le \alpha \le 1$, $0 < p < \gamma/\alpha$, and $f \in M_\alpha^p(X)$. Then for any $\varepsilon > 0$ there exist a function f_ε and an open set $O_\varepsilon \subset X$ such that*

1) $\mathrm{Cap}_{\alpha-\beta,p}(O_\varepsilon) < \varepsilon$, $\mathbb{H}_\infty^{\gamma-(\alpha-\beta)p}(O_\varepsilon) < \varepsilon$.

2) $f = f_\varepsilon$ *on* $X \setminus O_\varepsilon$,

3) $f_\varepsilon \in M_\alpha^p(X) \cap H^\beta(B)$ *for any ball* $B \subset X$,

4) $\|f - f_\varepsilon\|_{M_\alpha^p(X)} < \varepsilon$.

In the case $p \ge 1$ these results are mainly known (see [1] and references in this paper for $p > 1$, and [2] for $p = 1$). In the recent paper [3], our theorem is proved by another methods (without the statement about capacities $\mathrm{Cap}_{\alpha,p}$) for wider scales of Besov and Triebel–Lizorkin spaces.

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On the norms of the means of spherical Fourier sums

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The report deals with the spherical Fourier sums $S_r(f,x)=\sum\limits_{||k||\le r}\hat{f}(k)e^{ik\cdot x}$ of a periodic function f on m variables, the strong means

$$\left(\frac{1}{n}\sum_{j=0}^{n-1}\left|S_j(f,x)\right|^p\right)^{1/p}$$

and the strong integral means $\left(\left(\int\limits_0^R |S_r(f,x)|^p\,dr\right)/R\right)^{1/p}$ of these sums for $p\ge 1$. The extract growth orders as $n\to\infty$ and $R\to\infty$ of the corresponding operators, i.e., the growth orders of the quantities

$$\sup_{|f|\le 1}\left(\frac{1}{n}\sum_{j=0}^{n-1}\left|S_j(f,0)\right|^p\right)^{1/p} \quad \text{and} \quad \sup_{|f|\le 1}\left(\left(\int\limits_0^R |S_r(f,0)|^p\,dr\right)/R\right)^{1/p}$$

are established. The upper and lower bounds differ by their coefficients, which depend only on the dimension m.

A sufficient condition on the function ensuring the uniform strong p-summability of its Fourier series is given.

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On Sobolev and potential spaces related to Jacobi expansions

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We define and study Sobolev spaces associated with Jacobi expansions. We prove that these Sobolev spaces are isomorphic, in the Banach space sense, with potential spaces (for the Jacobi 'Laplacian') of the same order. This is an essential generalization and strengthening of the recent results [1] concerning the special case of ultraspherical expansions, where in addition a restriction on the parameter of type was imposed. We also present some further results and applications, including a variant

of Sobolev embedding theorem. Moreover, we give a characterization of the Jacobi potential spaces of arbitrary order in terms of suitable fractional square functions. As an auxiliary result of independent interest we prove L^p-boundedness of these fractional square functions.

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On a lower bound of periodic uncertainty constant

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Suppose $f \in L_2(\mathbb{T})$, then the functional $UC_B(f) := \sqrt{\mathrm{var}_{\mathrm{A}}(f)\,\mathrm{var}_{\mathrm{F}}(f)}$ is called the periodic (Breitenberger) uncertainty constant, where

$$\mathrm{var}_{\mathrm{A}}(f) := \frac{\|f\|^4}{|(\mathrm{e}^{i\cdot} f, f)|^2} - 1, \quad \mathrm{var}_{\mathrm{F}}(f) := \frac{\|if'\|^2}{\|f\|^2} - \frac{(if', f)^2}{\|f\|^4}.$$

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It is well known (Breitenberger, Prestin, Quak) that if f is not a trigonometric monomial, then $UC_B(f) > 1/2$ and there is no function such that $UC_B(f) = 1/2$.

In this talk we discuss an inequality refining the lower bound of the UC_B for a wide class of sequences of periodic functions. Namely, under some conditions we get $\lim_{j\to\infty} UC_B(\psi_j) \geq 3/2$ for a sequence $\psi_j \in L_2(\mathbb{T})$.

This work also has the following motivation. A family of periodic Parseval wavelet frames Ψ^a is constructed (Lebedeva, Prestin). The family has optimal time-frequency localization (the UC_B tends to 1/2) with respect to a family parameter, and it has the best currently known localization (the UC_B tends to 3/2) with respect to a multiresolution analysis parameter. Now it turns out that the family has optimal localization with respect to both parameters within the class of functions considered here. This class contains Ψ^a, periodic Parseval wavelet frames generated by periodization, and some practically important classes of general periodic Parseval wavelet frames.

On theorems of F. and M. Riesz

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We discuss various analogs of the famous theorem due to F. and M. Riesz on the absolute continuity of the measure whose negative Fourier coefficients are all zeros. A simpler and more direct proof of one of such analogues is obtained. In the same spirit a different proof is found for another theorem of F. and M. Riesz on absolute continuity. These results are closely related to one theorem of Hardy and Littlewood on the absolute convergence of the Fourier series of a function of bounded variation whose conjugate is also of bounded variation and its extensions to the non-periodic case. Certain multidimensional results are discussed as well.

Wavelets on local fields of characteristic zero

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Let K be a local field with non-Archimedean norm $\|x\|$, K^+ – an additive group of K, $\mathfrak{D} = \{x \in K: \ \|x\| \le 1\}$ – the ring of integers, $\mathfrak{B} = \{x \in K: \|x\| < 1\}$ – the maximal ideal in $\mathfrak{D}$, $\mathfrak{p}$ a prime element of K. The dilation operator $\mathcal{A}$ is defined by $\mathcal{A}x = x\frac{1}{\mathfrak{p}}$. Let $I_k = \{g = a_{-1}\mathfrak{p}^{-1}\dot{+}\ldots\dot{+}a_{-\nu}\mathfrak{p}^{-\nu} : \nu \in \mathbb{N}, a_j \in k\}$ be a set of dilations.

Only one example of MRA on the field of characteristic zero with $s > 1$ is known [1]. This MRA is generated by the function $\varphi(x) = \mathbf{1}_{\mathfrak{D}}(x)$. We will propose methods to construct another bases based on the following theorems.

Theorem 1. *The additive group $F^{(s)+}$ of the local field $F^{(s)}$ of characteristic zero is homeomorphic to the product $\mathbb{Q}_p^s$.*

From the theorem 1 we obtain

Theorem 2. *Let $K = F^{(s)}$ be a local field of characteristic zero,*

$$\mathfrak{p} = (\ldots, \mathbf{0}_0, (1, 0. \ldots, 0)_1, \mathbf{0}_2, \ldots).$$

Define the function $\hat{\varphi}(\xi)$ as

$$\hat{\varphi}(\xi) = \prod_{n=0}^{\infty} m_0(\xi\mathcal{A}^{-n}),$$

where mask $m_0(\xi)$ is constant on cosets $(K^+)^{\perp}_{-N}\chi$, $m_0((K^+)^{\perp}_{-N}) = 1$, $|m_0((K^+)^{\perp}_0)| = 1$, $m_0((K^+)^{\perp}_1 \setminus (K^+)^{\perp}_0) = 0$. Then φ generates an orthogonal MRA.

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Basis Properties of Generalised p-cosine Functions

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Consider a periodic function F, such that its restriction to the unit segment lies in the Banach space $L_s = L_s(0,1)$ for $s > 1$. Denote by S the family of dilations $F(nx)$ for all n positive integer. The purpose of this talk is to discuss the following question: When does S form a Schauder basis of L_s?

At first sight, one might think that this question has been studied considerably in the past, for instance, in the context of Paley-Wiener-type theorems. As it turns, this has not been the case, and the latter does not seem to be of much use in this respect.

We will formulate general criteria which apply to the particular case of F being the p-sine and the p-cosine functions. Both these functions arise naturally in the context of the non-linear eigenvalue problem associated to the one-dimensional p-Laplacian in the unit segment. Our main goal will be to determine a range of values for the parameter p, such that the dilated p-cosine functions form a Schauder basis of L_s. The case $s = 2$ (Riesz basis) will be examined with particular attention. Our results improve upon those from [Edmunds, Gurka, Lang, *J. Math. Anal. Appl.* 420 (2014)].

Continuability of multiple power series into sectorial domain by meromorphic interpolation of coefficients

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Consider a multiple power series

$$f(z) = \sum_k f_k z^k. \tag{1}$$

Following Ivanov [1] we introduce the following set

$$T_\varphi(\theta) = \{\nu \in \mathbb{R}^n : \ln|\varphi(re^{i\theta})| \leq \nu_1 r_1 + \ldots + \nu_n r_n + C_{\nu,\theta}\},$$

where the inequality is satisfied for any $r \in \mathbb{R}^n_+$ with some constant $C_{\nu,\theta}$.

Denote

$$T_\varphi := \bigcap_{\theta_j = \pm\frac{\pi}{2}} T_\varphi(\theta_1, \ldots, \theta_n),$$

$$M_\varphi := \{\nu \in [0,\pi]^n : \ \nu + \varepsilon \in T_\varphi, \ \ \nu - \varepsilon \notin T_\varphi \quad \text{for any} \ \ \varepsilon \in \mathbb{R}^n_+\}.$$

Let G be a sectorial set

$$G = \bigcup_{\nu \in M_\varphi} G_\nu, \tag{2}$$

i.e., a union of open polyarcs

$$G_\nu = (\mathbb{C} \setminus \Delta_{\nu_1}) \times \ldots \times (\mathbb{C} \setminus \Delta_{\nu_n})$$

where $\Delta_{\nu_j} = \{z = re^{i\vartheta} \in \mathbb{C} : |\theta| \leq \nu_j\}$.

Theorem. *The sum of the series (1) extends analytically to a sectorial domain G of the form (2) if there is an entire function $\varphi(\zeta)$ of exponential type*

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interpolating the coefficients f_k *and a vector-function* $\nu(\theta)$ *on* $[-\frac{\pi}{2}, \frac{\pi}{2}]^n$ *with values in* $M_\varphi(\theta)$ *satisfying*

$$\nu_j(\theta) \le a|\sin\theta_j| + b\cos\theta_j, \quad j = 1, ..., n,$$

with some constants $a \in [0, \pi)$, $b \in [0, \infty)$.

In the proof we use the theory of multiple residues and the principle of separating cycles that allows to represent an integral of a meromorphic form over the skeleton of a polyhedron by the sum of residues at some points in the polyhedron [2].

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Some Constructions of Grassmannian Fusion Frame

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When data is transmitted over communication channel, it might be corrupted by noise or lost. Frame is a useful representation of data because of redundancy. There are evidences that some frames work better than others. For instance, Grassmannian frame provides a representation which is robust against noise and multiple erasures. Grassmannian frame is characterized by the property that the maximal cross corellation of the frame elements have minimal value among a given class of frames. The definition of Grassmanian frame can be generalized to Grasmanian fusion frame which is a set of subspaces that the minimal chordal distance between subspaces has the maximal value among a given class of fusion frames. Similar to Grasmanian frame, Grasmannian fusion frame is robust against noise and multiple erasures. In more details, Calderbank et. al show that a fusion frame is optimally robust against noise if the fusion frame is tight and also a tight fusion frame is optimally robust against one subspace erasure if the dimensions of the subspaces are equal. They also proved that a tight fusion frame is optimally robust against multiple erasures if the subspaces are equidistance. Simultaneously being robust against noise and erasures, a representation of data with presence of a small number of sources are needed. Sparse fusion frame gives us such a representation.

This paper contains a new approach to construct an optimal Grassmannian fusion frame for various redundancies, along with some illustrative examples. Finally we impose some conditions on an algorithm by Calderbank et. al such that the output becomes a Grassmannian fusion frame.

Conserved Least-Squares Meshless Method for Two Dimensional Heat Transfer Solution

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A domain of heat transfer problem is discretized with distributed points and the least-square meshless method is used for the discretization of the two dimensional heat transfer equation. The spatial derivatives of the function by using the least-square method are as following:

$$\left(\frac{\partial T}{\partial x}\right)_i = \sum_{j=1}^{m} a_{ij}(T_j - T_i) \ , \ \left(\frac{\partial T}{\partial y}\right)_i = \sum_{j=1}^{m} b_{ij}(T_j - T_i). \tag{1}$$

Here T is the temperature and j is in a cloud of point i. The coefficients a_{ij}, b_{ij} are the least-square coefficients, and can be calculated as:

$$\begin{aligned} a_{ij} &= \frac{\omega_{ij}\Delta x_{ij}(\sum_{k=1}^{m}\omega_{ik}\Delta y_{ik}^2) - \omega_{ij}\Delta y_{ij}(\sum_{k=1}^{m}\omega_{ik}\Delta x_{ik}\Delta y_{ik})}{(\sum_{k=1}^{m}\omega_{ik}\Delta x_{ik}^2)(\sum_{k=1}^{m}\omega_{ik}\Delta y_{ik}^2) - (\sum_{k=1}^{m}\omega_{ik}\Delta x_{ik}\Delta y_{ik})^2}, \\ b_{ij} &= \frac{\omega_{ij}\Delta y_{ij}(\sum_{k=1}^{m}\omega_{ik}\Delta x_{ik}^2) - \omega_{ij}\Delta x_{ij}(\sum_{k=1}^{m}\omega_{ik}\Delta x_{ik}\Delta y_{ik})}{(\sum_{k=1}^{m}\omega_{ik}\Delta x_{ik}^2)(\sum_{k=1}^{m}\omega_{ik}\Delta y_{ik}^2) - (\sum_{k=1}^{m}\omega_{ik}\Delta x_{ik}\Delta y_{ik})^2}, \end{aligned} \tag{2}$$

where ω is an arbitrary weight function such as normalized Gaussian. The conservation law using the least-square method will be satisfied for a domain if the following relations are satisfied:

$$\sum_{j=1}^{m}\omega_{ij}\Delta x_{ij}^2 = \sum_{j=1}^{m}\omega_{ij}\Delta y_{ij}^2 \ , \ \sum_{j=1}^{m}\omega_{ij}\Delta x_{ij}\Delta y_{ij} = 0. \tag{3}$$

In general, the constraints of Eq. (3) are not satisfied except in some simple configurations, such as uniform Cartesian spacing. However, the constraints can be satisfied on arbitrary point distributions as part of a local weight optimization procedure. In the optimization problem, the cost function is defined by requiring that the sum of the squares of the perturbations in weights is minimal:

$$C_i(\overline{\omega}) = \sum_{j=1}^{m} \delta\omega_{ij}^2 \ \ , \ \ \delta\omega_{ij} = \omega_{ij} - \overline{\omega}_{ij} \tag{4}$$

subject to the constraints

$$h_i(\overline{\omega}) = \sum_{j=1}^{m} \overline{\omega}_{ij}(\Delta x_{ij}^2 - \Delta y_{ij}^2) \ \ , \ \ q_i(\overline{\omega}) = \sum_{j=1}^{m} \overline{\omega}_{ij}\Delta x_{ij}\Delta y_{ij}. \tag{5}$$

We take the standard approach of Lagrange multipliers and write the unconstrained problem as follows:

$$\min\,[C_i(\overline{\omega}) + \sigma_1 h_i(\overline{\omega}) + \sigma_2 q_i(\overline{\omega}).] \tag{6}$$

The method of Lagrange multipliers may be invoked here, yielding the following system of equations at each node:

$$\begin{bmatrix} \mathbf{I} & \mathbf{U}^T \\ \mathbf{U} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \overline{\omega} \\ \boldsymbol{\text{œ}} \end{bmatrix} = \begin{bmatrix} \omega \\ \mathbf{0} \end{bmatrix} \tag{7}$$

where

$$\mathbf{U} = \begin{bmatrix} (\Delta x_{i1}^2 - \Delta y_{i1}^2) & (\Delta x_{i2}^2 - \Delta y_{i2}^2) & \ldots & (\Delta x_{im}^2 - \Delta y_{im}^2) \\ \Delta x_{i1}\Delta y_{i1} & \Delta x_{i2}\Delta y_{i2} & \ldots & \Delta x_{im}\Delta y_{im} \end{bmatrix} \tag{8}$$

and $\boldsymbol{\text{œ}} = [\sigma_1 \ \ \sigma_2]$ is the vector of Lagrange multipliers. This system of equations is independent to the unknown variable and is solved on a local node by node basis once in the beginning of the simulation. Therefore, the least-square coefficients with modified weights will be as follows:

$$a_{ij} = \frac{\omega_{ij}\Delta x_{ij}}{\sum_{k=1}^{m} \omega_{ik}\Delta x_{ik}^2} \, , \, b_{ij} = \frac{\omega_{ij}\Delta y_{ij}}{\sum_{k=1}^{m} \omega_{ik}\Delta y_{ik}^2}. \tag{9}$$

Above relations for least-squares coefficients are used to solve the governed heat transfer equation.

Generic boundary behaviour of Taylor series in Hardy and Bergman spaces

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This is a joint work with Hans-Peter Beise.

It is known that, generically, Taylor series of functions holomorphic in the unit disc turn out to be universal series outside of the unit disc and in particular on the unit circle. Due to classical and recent results on the boundary behaviour of Taylor series, for functions in Hardy spaces and Bergman spaces the situation is essentially different. In this talk it is shown that in many respects these results are sharp in the sense that universality generically appears on maximal exceptional sets.

Bernstein-type inequalities on Jordan arcs

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Bernstein-type polynomial inequalities are well known and frequently used. On the complex plane a Bernstein-type inequality was proved by Nagy-Totik (2005) for compact sets bounded by finitely many, smooth Jordan curves. The result is asymptotically sharp and its formulation uses the normal derivative of Green's function. Jordan arcs present additional difficulties. The corresponding inequality was found for general subsets of the unit circle by Nagy-Totik in 2013 and 2014. In the talk the analogous result for smooth Jordan arcs will be discussed for both polynomials and rational functions. The approach uses open-up technique, Gonchar-Grigorjan type estimates, some results on Faber operators and Borwein-Erdélyi inequality for rational functions. Some open problems and conjecture will also be mentioned.

Universal functions in a sense of modification with respect to Fourier coefficients

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It is proved that there exists a function $g(x) \in L^1[0,1]$ with monotonically decreasing Fourier-Walsh coefficients $\{c_n(g)\}_{n=0}^{\infty}$, such that for every function $f \in L^p[0;1]$ and for any $\varepsilon > 0$ there exists a function $\tilde{f} \in L^p[0;1]$ whose Fourier series with respect to the Walsh system converges to $\tilde{f}(x)$ in $L^p[0,1]$ norm, Fourier-Walsh coefficients satisfy

$$|c_k(\tilde{f})| = c_k(g), \qquad k \in Spec(\tilde{f})$$

and

$$mes\{x \in [0;1] : f(x) = \tilde{f}(x)\} > 1 - \varepsilon.$$

It is also proved that for any $0 < \varepsilon < 1$ there exist a measurable set $E \subset [0,1]$, with measure $mesE > 1 - \varepsilon$, and a function $g \in L^1[0;1]$, with Fourier-Walsh coefficients satisfying

$$0 < c_{k+1}(g) < c_k(g), \qquad k = 0, 1, 2, ...,$$

such that for any function $f \in L^1[0,1]$ there exists a function $\tilde{f} \in L^1[0,1]$, coinciding with f on E, whose Fourier-Walsh series converges to $\tilde{f}(x)$ in $L^1[0,1]$ norm, and the Fourier-Walsh coefficients satisfy

$$|c_k(\tilde{f})| = c_k(g), \qquad k = 0, 1, 2, ...$$

Recognition of convex bodies by probabilistic methods

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The main purpose of the stereology is to obtain information about the geometric properties of n-dimensional structures, if there is an information on the forms of smaller dimensions as through the k-flats (k-dimensional planes) sections ($0 < k \leq n-1$), and with the help of projections on infinitesimal layers. The most popular application is the tomography (see [1] and [2]). Reconstruction of a body over its cross sections is one of the main tasks of geometric tomography, a term introduced by R. Gardner in [1]. If $D \subset \mathbf{R}^n$ ($\mathbf{R}^n$ is the n-dimensional Euclidean space) is intersected by k-plane, then arises a k-dimensional section that contains some information on D. A natural question arises whether it is possible to reconstruct D, if we have a subclass of k-dimensional cross-sections. The recognition of bounded convex bodies D by means of random k-flats intersecting D is one of the interesting problems of Stochastic Geometry. In particular, the problem of recognition of bounded convex domains D by chord length distribution function is of much interest (see [3]). One can consider the case when the orientation and the length of the chords are observed. We refer this case as the orientation-dependent chord length distribution. All these problems are the problems of geometric tomography (see [1]), since orientation-dependent chord length distribution function at point y is the probability that parallel X-ray in a fixed direction is less than or equal to y. Investigation of convex bodies by orientation-dependent chord length distribution is equivalent to the investigation of their covariograms. The present talk considers some problems and recent results related to covariograms, and their applications to various problems of tomography [4]— [7].

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Spectral properties of measures

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In this talk I would like to present some recent results concerning the Banach algebra $M(\mathbb{T})$ of Borel regular measures on the circle group with the convolution product. Since it is well-known that the spectrum of a measure can be much bigger than the closure of the values of its Fourier-Stieltjes transform (the Wiener-Pitt phenomenon) it is natural to ask what kind of topological properties of the Gelfand space $\mathfrak{M}(M(\mathbb{T}))$ are responsible for this unsual spectral behaviour. It follows immediately from the existence of the Wiener-Pitt phenomenon that the set $\mathbb{Z}$ identified with Fourier-Stieltjes coefficients is not dense in $\mathfrak{M}(M(\mathbb{T}))$. However, it is not clear if any other countable dense subset of this space exists. During my talk, I will disprove this fact - i.e. I will show the non-separability of the Gelfand space of the measure algebra on the circle group. This result is contained in the paper 'On topological properties of the measure algebra on the circle group' written in a collaboration with Micha Wojciechowski which has not been published yet, but is available on arxiv.org with identifier: 1406.0797.

Rotation of Coordinate Axes and Differentiation of Integrals with respect to Translation Invariant Bases

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The dependence of differentiation properties of an indefinite integral on a rotation of coordinate axes is studied, namely: the result of J. Marstrand on the existence of a function the integral of which is not strongly differentiable for any choice of axes is extended to Busemann-Feller and homothecy invariant bases which does not differentiate $L(\mathbb{R}^n)$; it is proved that for an arbitrary translation invariant basis B formed of multi-dimensional intervals and which does not differentiate $L(\mathbb{R}^n)$, the class of all functions the integrals of which differentiate B is not invariant with respect to rotations, and for bases of such type it is studied the problem on characterization of singularities that may have an integral of a fixed function for various choices of coordinate axes.

This is a joint work with K.Chubinidze.

Rational series and operators in the theory of approximation

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The study of Fourier series for Takenaka – Malmquist orthogonal system of rational functions with prescribed poles was initiated by M. M. Dzhrbashyan [1]. In particular, in [1] a compact expression for the Dirichlet kernel for that orthogonal system was found, analogues of Dirichlet – Jordan and Dini – Lipschitz tests for the trigonometric system were proven. Dzhrbashyan's ideas received development in different directions. For example, G.S. Kocharyan obtained an estimate of the Lebesgue constants of the Takenaka – Malmquist system (see [3]). M.M. Dzhrbashyan and G. Tumarkin have built a system of rational functions, which generalize the Faber polynomials (see [4]). The study of these systems was continued in works of A.M. Lukatski, A.A. Kitbalyan and others.

Another direction of the research initiated by Dzhrbashyan was launched in Belarus. Based on the results of [1], V.N. Rusak (see [5]) built rational operators of Fejér, Jackson and Valle–Poussin type and studied their approximation properties. Rational operators, introduced by V.N. Rusak, are widely used in rational approximation with both fixed and with free poles (see [5, 6]). In our report we are going to consider some of the results of such type.

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Duality and bounded projections in spaces of analytic or harmonic functions

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The duality problem in spaces of functions analytic in the unit ball of $\mathbb{C}^n$ and harmonic in the unit ball of $\mathbb{R}^n$, $n > 2$, is investigated.

We use the same approach to the duality problem as [1]. This approach depends on showing that a certain integral operator from L^∞ to analytic or, respectively, harmonic subspace is a bounded projection. The kernel of the integral operator is the reproducing kernel.

The multidimensional case has the specifics in the sense that there is no connection between harmonic and holomorphic functions, that is, not every harmonic function is the real part of an analytic function.

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Dyadic measures and uniqueness problems for Haar series

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The paper is devoted to uniqueness problems for one- and multidimensional series by the Haar system of functions.

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It is well known that any Haar series can be represented as a quasi-measure, i.e. a finitely additive set function defined on the set of all dyadic intervals. The behavior of the partial sums and convergence of Haar series often may be described in terms of continuity, or smoothness, or differentiability of the corresponding quasi-measures. (See, for example, [1, 2, 3, 4] about the correspondence between Haar series and quasi-measures.)

Let $\{f_n\}$ be a system of function defined on some set X. Recall that a set $A \subset X$ is said to be a *set of uniqueness* (or $\mathcal{U}$*-set*) for series $\sum_n a_n f_n(x)$, $a_n \in \mathbb{R}$ or $\mathbb{C}$, if the only series converging to zero on $X \setminus A$ is the trivial series.

In some papers of the author (see, for instance, [5]) it has been shown, for some classes of Haar series, that the fact of belonging of a closed set $A \subset [0,1]$ to the family of $\mathcal{U}$-sets is equivalent to the existence of non-trivial quasi-measure having the definite order of smoothness.

We intend to discuss some properties of quasi-measures and dyadic measures, and related results concerning U-sets for Haar series.

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Asymptotic Estimates for quasi-periodic Interpolations

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We consider the problem of function interpolation by the quasi-periodic interpolants $I_{N,m}(f,x)$, $m \geq 0$ (m is an integer), $x \in [-1,1]$, which interpolate f on equidistant grid

$$x_k = \frac{k}{N}, \; |k| \leq N$$

and is exact for quasi-periodic functions

$$e^{i\pi n\sigma x}, \; |n| \leq N, \; \sigma = \frac{2N}{2N+m+1}$$

with the period $2/\sigma$, which is greater than the length of the interval, but tends to that length as the number of nodes grows to infinity ([1]).

First, we study the pointwise convergence of the quasi-periodic interpolations and derive exact constants for the asymptotic errors showing fast convergence compared to the classical interpolations (see [2]).

Second, we study ([3], [4]) the L_2-convergence of the quasi-periodic interpolations and also their behavior at the endpoints of the interval in terms of the limit functions. In both cases, we derive exact constants of asymptotic errors and compare them with the classical analogues.

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On the convergence of Cesaro means of Walsh series in $L^p[0,1]$, $p > 0$

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Let

$$\sum_{k=0}^{\infty} a_k W_k(x) \tag{1}$$

be a series in the Walsh system $\{W_k(x)\}_{k=0}^{\infty}$.

The talk is devoted to the convergence of Cesaro means of series (1)

$$\sigma_n^{\alpha}(x) = \frac{1}{A_n^{\alpha}} \sum_{m=0}^{n} A_{n-m}^{\alpha-1} S_m(x) \quad (\alpha > -1),$$

where

$$A_n^{\alpha} = \frac{(\alpha+1)(\alpha+2)\cdots(\alpha+n)}{n!}$$

and

$$S_m(x) = \sum_{k=0}^{m} a_k W_k(x),$$

in spaces $L^p[0,1],\ p > 0$.

On the divergence of Fourier-Walsh series of continuous function

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It is know that a continuous function may have diverging Fourier series with respect to the Walsh system (see [1]). Moreover, the following theorems are true

Theorem 1. *For any perfect set of positive measure $P \subset [0,1)$, and for it's any density point x_0, one can find a continuous function $f(x)$ on $[0,1)$, having the following property: any measurable function $g(x)$, bounded on $[0,1)$ and coinciding with $f(x)$ on P, has diverging Fourier-Walsh series at x_0.*

Theorem 2. *For any perfect set $P \subset [0,1)$ and for any point $x_0 \in [0,1)$ satisfying the condition $\int\limits_{[0,1)\setminus P} \frac{dx}{|x-x_0|} < \infty$ one can find a continuous function $f(x)$ on $[0,1)$ with $\sup\limits_{x:|x-x_0|<\delta} |f(x) - f(x_0)| = \frac{o(1)}{\log 1/\delta}$ $(\delta \to 0)$ and having the following property: any measurable function $g(x)$, bounded on $[0,1)$ and coinciding with $f(x)$ on P, has diverging Fourier-Walsh series at x_0.*

Note that the analogue of Theorem 1 in case of the trigonometric system is proved by D.E. Menshov (see [2]).

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Wavelets Associated with Nonuniform Multiresolution Analysis on Local Fields of Positive Characteristic

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Multiresolution analysis is considered as the heart of wavelet theory. The concept of multiresolution analysis provides a natural framework for understanding and constructing discrete wavelet systems. A generalization of Mallat's classic theory of multiresolution analysis on local fields of positive characteristic was considered by Jiang, Li and Jin [*Multiresolution analysis on local fields*, J. Math. Anal. Appl. 294 (2004), pp. 523-532]. In this paper, we present a notion of nonuniform multiresolution analysis on local field K of positive characteristic. The associated subspace V_0 of $L^2(K)$ has an orthonormal basis, a collection of translates of the scaling function φ of the form $\{\varphi(x-\lambda)\}_{\lambda\in\Lambda}$, where $\Lambda = \{0, r/N\} + \mathcal{Z}, N \geq 1$ is an integer and r is an odd integer such that r and N are relatively prime and $\mathcal{Z} = \{u(n) : n \in \mathbb{N}_0\}$. We establish a necessary and sufficient condition for the existence of associated wavelets and derive an algorithm for the construction of nonuniform multiresolution analysis on local fields starting from a refinement mask $m_0(\xi)$ with appropriate conditions. Moreover, we describe wavelets in the nonuniform discrete setting and provide a characterization of an orthonormal basis for $l^2(\Lambda)$. Further, our results also hold for Cantor dyadic group and Vilenkin p-adic groups.

Character amenability of dual Banach algebras

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In this paper we introduce the concept of character Connes-amenability for dual Banach algebras. Then the properties of dual Banach algebras with this new concept are investigated. Next, we try to get the equivalent conditions for character Connes-amenability of dual Banach algebras. We prove that left character amenability of $\mathcal{A}$ is equivalent to character Connes-amenability of $\mathcal{A}^{**}$ when $\mathcal{A}$ is Arens regular. Moreover for a locally compact group G, we show that $M(G)$ is always character Connes-amenable. In addition, by means of some example, we show that for the new notion, the corresponding class of dual Banach algebras is larger than Connes-amenability of dual Banach algebras.

Wavelets and Cubature Formulas

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Two approaches to the construction of optimal cubature formulae are considered. The approximation subspace is the span of lattice translations of the fixed function. This problem is closely associated with the search of characteristics of the best projection-net approximations. For example, in

some cases the optimal lattice satisfies the following condition: the dual lattice generates the densest packing of Lebesgue sets of some function depending on the norm of Hormander spaces (for Sobolev spaces the problem comes to the densest lattice packing of spheres).

Fourier transform on $CV_2(K)$

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Let G be a locally compact group. A convolution operator T on G is a linear operator on complex functions $\phi : G \to \mathbb{C}$ that commutes with left translations.

In this paper we extend basic definitions about convolution operators on locally compact hypergroups. Roughly speaking, a hypergroup K is a locally compact space if there is a convolution on the probability measures in $M(K)$ satisfying certain conditions.

Here also we give some results about an extension of Fourier transform on $CV_2(K)$, a special subalgebra of Banach algebra of all continuous linear endomorphisms on $L^p(K)$.

On some uniqueness problems of trigonometric series

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We discuss some uniqueness problems of trigonometric series and series in Haar system. Using the obtained results, a problem posed by P. L. Ulyanov in 1964 is investigated.

From Thresholding Greedy Algorithm to Chebyshev Greedy Algorithm

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The fundamental question of nonlinear approximation is how to devise good constructive methods (algorithms) of nonlinear approximation. In the case of nonlinear approximation with respect to a basis the Thresholding Greedy Algorithm is the simplest and the most studied one. The following question is very natural and fundamental. Which bases are suitable for the use of the Thresholding Greedy Algorithm (TGA)? Answering this question researchers introduced several new concepts of bases of a Banach space X: greedy bases, quasi-greedy bases, almost greedy bases. The greedy bases are the best for application of the TGA for sparse approximation - for any $f \in X$ the TGA provides after m iterations approximation with the error of the same order as the best m-term approximation of f. If a basis Ψ is a quasi-greedy basis then it merely guarantees that for any $f \in X$ the TGA provides approximants that converge to f but does not guarantee the rate of convergence. It turns out that the wavelet type bases are very good for the TGA. However, it is known that the TGA does not work well for the trigonometric system.

It was discovered recently, that the Weak Chebyshev Greedy Algorithm (WCGA) works much better than the TGA for the trigonometric system. We discuss and compare approximation by the TGA and the WCGA. We present some Lebesgue-type inequalities for the Weak Chebyshev Greedy Algorithm. The main message of the talk is that it is time to conduct a deep and thorough study of the WCGA with respect to bases in a style of the corresponding study of the TGA.

On the maximal operators of Vilenkin-Norlund means on the martingale Hardy spaces

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The Nörlund summation is a very general summability method. It is well-known in the literature that the so-called Nörlund means are generalizations of the Fejér, Cesáro and logarithmic means. Therefore, it is of prior interest to study the behavior of operators related to Nörlund means of Fourier series with respect to orthonormal systems.

This lecture is devoted to review the maximal operators of Nörlund means on the martingale Hardy spaces (for the details concerning to martingale Hardy spaces see e.g. Weisz [3, 4]). In particular, we discuss some new $\left(H_p, weak-L_p\right)$ and $\left(H_p, L_p\right)$ type inequalities of maximal operators of Vilenkin-Nörlund means with monotone coefficients. These results are the best possible in a special sense. As applications, both some well-known and new results are pointed out. For example, by applying these results we can conclude a.e. convergence of such Vilenkin-Nörlund means.

The talk is based on joint works with co-authors Nacima Memić, Lars-Erik Persson and Peter Wall.

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On some factorization properties of poised and independent sets of nodes

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An n-poised set in two dimensions is a set of nodes admitting unique bivariate interpolation with Π_n : the space of polynomials of total degree at most n. If a set $\mathcal{X}$ is n-poised then $\#\mathcal{X} = N := \dim \Pi_n = (1/2)(n+2)(n+1)$. The subsets of n-poised sets are called n-independent sets. These are exactly sets with which interpolation is solvable (not necessarily uniquely).

Theorem 1. *Assume that $\mathcal{X}$ is an n-poised set of nodes and a line ℓ passes through exactly three nodes. Assume also that ℓ divides the fundamental polynomials of nodes of $\mathcal{X}_1 \subset \mathcal{X}$. Then $\#\mathcal{X}_1 \leq 3$. Moreover, we have equality here if and only if there is a curve of degree $n-2$ passing through all $N-6$ nodes of $\mathcal{X} \setminus (\ell \cup \mathcal{X}_1)$.*

This theorem is proved in [1].

Set $p|_{\mathcal{X}}$ for the restriction of p on $\mathcal{X}$. We have also

Theorem 2. *Assume that ℓ is a line and $\mathcal{X}_1 \subset \ell$ is a set of k nodes, where $k = 2, 3$. Assume also that a set of nodes $\mathcal{X}_2$ is given with $\mathcal{X}_2 \cap \ell = \emptyset$ such that*

$$p \in \Pi_n,\ p|_{\mathcal{X}_1} = 0,\ p|_{\mathcal{X}_2} = 0 \ \Rightarrow\ p|_{\ell} = 0.$$

Then we have that $\#\mathcal{X}_2 \geq N - (1/2)k(k+1)$. Moreover, in the case of equality here the node set $\mathcal{X}_1 \cup \mathcal{X}_2$ is n-independent and there is a curve of degree $n-k+1$ passing through all the nodes of $\mathcal{X}_2$.

We think that this theorem holds for the general k.

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Fast decreasing polynomials at corners

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Fast decreasing polynomials appear in many different situations for they are particularly useful in localization. On the real line Ivanov and Totik have given a comprehensive characterization of these polynomials summing up and extending the earlier results [1]. On curves, however, our knowledge is narrow: although some results can be transformed from interval over curves (e.g. [2, Theorem 4.1]), but these prove sharp only in the case of smooth curves.

In our talk we discuss how one can construct fast decreasing polynomials at Dini-smooth corners. Besides, we investigate the question how the rate of decay depends on the angle at the corner. We also mention some consequences including Nikolskii and Bernstein type inequalities for area measures.

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Numerical integration, Haar projection numbers and failure of unconditional convergence

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Efficient approximation and integration of multivariate functions is a crucial task for the numerical treatment of many multi-parameter real-world problems. In the last 50 years several methods for the efficient numerical integration of multivariate functions from Sobolev, Nikol'skij and Korobov classes have been studied, among those are Korobov's lattice rules, Smolyak cubature formulas, Frolov's method and quasi-Monte Carlo rules based on digital nets. It turned out that choosing suitable points in a d-dimensional domain is connected with deep problems in number theory. Here we are interested in optimal (in order) methods for the numerical integration of d-variate functions with a bounded mixed derivative. In the first part of the talk we present (modern) QMC rules which are optimal for classes with smoothness less than two. We further comment on cubature on Smolyak grids. Those methods are able to exploit arbitrary high mixed smoothness. However, we show that that can never perform asymptotically optimal. And finally, we present recent results for Frolov's method which provides both, universality and optimality.

It turns out that also Frolov's method shows an interesting and unusual behavior for classes with small mixed smoothness. Such an effect was known for the Fibonacci lattice in $d = 2$ (Temlyakov). Apparently, this behavior is connected to the "absence" of unconditional convergence of Faber and Haar basis expansions in certain Sobolev spaces with small mixed smoothness. Indeed, in the second part of the talk we will present recent results on Haar projection numbers in such a framework (also for more general Triebel-Lizorkin spaces). Our estimates rely on probabilistic arguments and quantify the failure of unconditional convergence of Haar basis expansions. It has been an open question whether or not the Haar basis represents an unconditional basis in Sobolev spaces $W_p^s(\mathbb{R})$ with $1 < p < 2$ and $1/2 \leq s \leq 1/p$. We answer this question negatively. The talk is based on joint works with Dinh Dũng, Andreas Seeger and Mario Ullrich.

Optimal uniform approximation on $\mathbb{R}$ by harmonic functions on $\mathbb{R}^2$

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In this talk we discuss the problem of the uniform approximation on $\mathbb{R}$ by harmonic functions with an estimate of the growth of approximating functions.

The analogous problem in the case of entire functions was investigated by Arakelian [1], and in the case of meromorphic functions it was studied by Arakelian and Avetisyan [2].

In [3] the approximation was realized by harmonic functions on the given strip, and the following result on the growth of u on S_{2h} was obtained:

Theorem 1. *Let $f \in C^3(\mathbb{R})$ and $\varepsilon > 0$. Then for $h > 0$ there is a function $v \in h(S_h)$, satisfying*

$$|f(x) - v(x)| < 5\varepsilon \text{ for } x \in \mathbb{R},$$

the growth of which for $z = x + iy \in S_h$ is restricted by the inequality

$$|v(z)| < 3m_f(x,h) + 3\varepsilon \exp\left(5 + 2\sqrt{(2h)^3 \varepsilon^{-1} \omega_{1,f^{(3)}}(x,h)}\right).$$

In this talk we will talk about the approximation of the given function f by functions w harmonic on $\mathbb{R}^2$.

Theorem 2. *Let $f \in C^3(\mathbb{R})$ and $\varepsilon > 0$. Then there is a function w harmonic on $\mathbb{R}^2$ satisfying*

$$|f(x) - w(x,0)| < \varepsilon \text{ for } x \in \mathbb{R},$$

the growth of which for $(x,y) \in \mathbb{R}^2$ is restricted by the inequality

$$\ln|w(x,y)| < c \ln m_f(r+2) + c \ln r + c\varepsilon\sqrt{\varepsilon^{-1}\omega_{1,f^{(3)}}(x,1)}$$

where $r = \sqrt{x^2 + y^2}$ and $c > 0$ is an absolute constant.

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Quasi-greedy bases in Hilbert and Banach spaces

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It is known that an unconditional basis is quasi-greedy. It is also known that the quasi-greedy basis need not be unconditional. I will present various examples and theorems that explain the relation between quasi-greedy and unconditional in detail.

Gordon's Conjectures: Pontryagin-van Kampen Duality and Fourier Transform in Hyperfinite Ambience

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Using the ideas of E. I. Gordon [1], [2] we present an approach, based on *nonstandard analysis* (NSA), to simultaneous approximation of locally compact abelian (LCA) groups and their duals by finite abelian groups, as well as to approximation of the Fourier transforms on various functional spaces over them by the discrete Fourier transform (DFT). In 2012 we proved the three Gordon's Conjectures (GC1–3) which were open since 1991 and are crucial both in the formulations and proofs of the LCA groups and Fourier transform approximation theorems. The proofs of GC1 and GC2 combine some methods of NSA with Fourier-analytic methods of *additive combinatorics*, stemming from the paper [3] by Green and Ruzsa and the book [4] by Tao and Vu. The proof of GC3 relies on a fairly general nonstandard version of the *Smoothness-and-Decay Principle*.

Our approach is based on representing LCA groups by triplets (G, G_0, G_{f}), where G is a hyperfinite abelian group, and $G_0 \subseteq G_{\mathrm{f}}$ are its external subgroups, intuitively viewed as the monad of infinitesimal elements and the galaxy of finite elements, respectively. It can be shown that every LCA group $\mathbf{G}$ is isomorphic to the *observable trace* or *nonstandard hull* $G^\flat = G_\mathrm{f}/G_0$ of such a triplet. The dual triplet of (G, G_0, G_f) is defined as $\big(\widehat{G}, G_\mathrm{f}^{\perp\!\!\perp}, G_0^{\perp\!\!\perp}\big)$, where $\widehat{G}$ is the group of all internal homomorphisms (characters) $G \to {}^*\mathbb{T}$, and the *infinitesimal annihilators*

$$G_\mathrm{f}^{\perp\!\!\perp} = \big\{\chi \in \widehat{G} \mid \forall\, a \in G_\mathrm{f} : \chi(a) \approx 1\big\}\,,$$
$$G_0^{\perp\!\!\perp} = \big\{\chi \in \widehat{G} \mid \forall\, a \in G_0 : \chi(a) \approx 1\big\}$$

of the subgroups G_f, G_0 consist of characters which are infinitesimally close to 1 on the galaxy G_f or continuous in the intuitive sense backed by NSA, respectively.

GC1 states that for $\mathbf{G} = G^\flat$ its dual group $\widehat{\mathbf{G}}$ is canonically isomorphic to the observable trace $\widehat{G}^\flat = G_0^{\perp\!\!\perp}/G_\mathrm{f}^{\perp\!\!\perp}$ of the dual triplet $\big(\widehat{G}, G_\mathrm{f}^{\perp\!\!\perp}, G_0^{\perp\!\!\perp}\big)$. It turns out to be equivalent to the *Triplet Duality Theorem* according to which the dual triplet $\big(G, G_0^{\perp\!\!\perp\perp\!\!\perp}, G_\mathrm{f}^{\perp\!\!\perp\perp\!\!\perp}\big)$ of the dual triplet $\big(\widehat{G}, G_\mathrm{f}^{\perp\!\!\perp}, G_0^{\perp\!\!\perp}\big)$ coincides with the original triplet (G, G_0, G_f).

GC2 states certain natural duality relation, partly akin to the *Uncertainty Principle*, between "normalizing coefficients" or "elementary charges" on both the triplets, by means of which the Haar measures on their nonstandard hulls can be defined using the *Loeb measure* construction.

Representing the pair of dual LCA groups $\mathbf{G}$, $\widehat{\mathbf{G}}$ by a pair of dual triplets enables to approximate the Fourier-Plancherel transform $\mathrm{L}^2(\mathbf{G}) \to \mathrm{L}^2\big(\widehat{\mathbf{G}}\big)$ by means of the hyperfinite dimensional DFT ${}^*\mathbb{C}^G \to {}^*\mathbb{C}^{\widehat{G}}$. GC3 states that such an approximation is infinitesimally precise almost everywhere. Essentially the same is true also for the Fourier transform $\mathrm{L}^1(\mathbf{G}) \to \mathrm{C}_0\big(\widehat{\mathbf{G}}\big)$ and even for the Fourier-Stieltjes transform $\mathrm{M}(\mathbf{G}) \to \mathrm{C}_\mathrm{bu}\big(\widehat{\mathbf{G}}\big)$ extending it, as well as for the generalized Fourier transforms $\mathrm{L}^p(\mathbf{G}) \to \mathrm{L}^q\big(\widehat{\mathbf{G}}\big)$, for any pair of adjoint exponents $1 < p \le 2 \le q < \infty$.

Standard interpretations of these results imply the existence of "arbitrarily good" approximations of all the above Fourier transforms on every LCA group $\mathbf{G}$ by the DFT on some finite abelian group G.

Depending on time, we will survey most of the above mentioned constructions and results.

References

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